

# Non-linear filtration models and the effect of nonlinearity parameters on flow rates in low-permeability reservoirs

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**Abstract.** Filtration of oil in low-permeable reservoirs is considered. The experimental data of dependence of filtration velocity on pressure gradient are analysed. It is shown that the filtration law in low-permeability reservoirs differs from the linear Darcy's law and from the non-linear law with an initial pressure gradient.

The power law of filtration in low-permeability reservoirs is experimentally substantiated. Models of nonlinear filtration influence on flow rate are proposed. The analysis of influence of nonlinear filtration parameters on flow rate in technogenically modified near-wellbore zone is carried out.

**Keywords:** low-permeability reservoirs, permeability, nonlinear filtration law, near-wellbore zone, flow rate

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## Introduction

At present, the structure of hydrocarbon reserves is deteriorating all over the world. Every year the search and development of low-permeability and ultra-low-permeability reservoirs becomes an increasingly urgent task.

Low-permeability reservoirs are distinguished by the boundary values of the absolute permeability. Various researchers give different boundary values ( $0.5\text{--}5.0$ )  $10^{-3}$   $\mu\text{m}^2$  (Fei et al., 2008; Xiong et al., 2009; Mikhailov, 2008; Levorsen, 1967; Wang et al., 2011).

Finding productive low-permeability reservoirs is most likely at great depths, where the pore space is exposed to high loads. Sampling results are the only reliable criterion for detecting low-permeability reservoirs. However, the sampling efficiency depends on the features of filtration in low-permeability reservoirs as well as on the degree of change in natural filtration properties in the near-wellbore zones.

The nature and law of filtration in low-permeability reservoirs differ significantly from the nature of filtration in high-permeability reservoirs due to the fact that small-radius capillaries are present in low-permeability reservoirs, they are characterized by a complex structure and high filtration resistance. Molecular forces acting

between the liquid and solid phases are of great importance. In the case of two-phase filtration in a low-permeability formation, higher values of capillary forces are observed due to the small radius of the pores, and this greatly affects the filtration. If the displacement pressure is insufficient to overcome the resistance of capillary and molecular forces, then the oil movement will be interrupted, it will split into droplets, this will lead to an increase in the filtration resistance and a decrease in the displacement effect.

The specificity of filtration in low-permeability reservoirs is the strong influence of interfacial interactions between filtration fluids and pore surface. Low-permeability terrigenous reservoirs has increased content of clay minerals with increased surface activity. The specificity of low-permeability reservoirs is an increase in the proportion of micro-pores, an increase in the difference in pore sizes between micro- and macro-pores, as well as an increase in specific surface area with a decrease in permeability. This specificity leads to anomalous manifestations of the interaction forces between fluids and the rock skeleton during filtration, which causes nonlinear filtration effects.

For standard reservoirs, the effects of interphase interactions are negligible, and their influence is not reflected in the laws of filtration. Therefore, for them, the filtration law for the moving phase corresponds to the linear Darcy's law.

$$V = \frac{k}{\mu} grad p \quad (1)$$

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where  $V$  is filtration rate of the moving phase,  $k$  is a classical formation permeability,  $\mu$  is a fluid viscosity.

Numerous data show that in low-permeability reservoirs, due to the interaction of the inter pore surface and liquid, an oil boundary layer appears on the inter pore surface (Markhasin, 1977). In the boundary layer, the composition and properties of oil are very different from the properties of mobile oil (Markhasin, 1977). The boundary layer of adsorbed oil causes hydrophobization of the inter pore surface, which leads to a change in wettability (Mikhailov et al., 2019). In laboratory studies, hydrophobization is reliably recorded even when oil is filtered through hydrophilic rocks (Mikhailov et al., 2019).

Therefore, in a low-permeability reservoir, oil is not a strictly Newtonian fluid; moreover, it cannot remain unchanged throughout the entire pore system. However, in most cases, for medium and high permeability formations, moving oil is considered as a Newtonian fluid. Applying Darcy's law, numerous problems of medium and high permeability formations development are successfully solved on the basis of the linear filtration theory. In medium and high permeability reservoirs, the boundary layer of oil is small, and the ratio of the boundary layer volume to the total pore volume is also small. Accordingly, the influence of the non-Newtonian behavior of oil in the boundary layer does not affect the filtration law, which remains linear. In low-permeability reservoirs, on the contrary, these effects cannot be ignored; they cause a deviation from the linear Darcy's law.

Numerous experiments conducted since the middle of the twentieth century have shown that filtration in low-permeability reservoirs does not obey the linear Darcy's law (Baikov et al., 2013; Li Xuanzhan, 2015; Baoquan et al., 2011; Hao et al., 2008; Wang et al. al., 2011). The dependence of the filtration rate on the pressure gradient is described by a nonlinear function, with the selection of the initial (starting) pressure gradient.

The traditional approximation of the filtration rate relation with pressure gradient for low-permeability reservoirs shows a very large value of the initial pressure gradient that triggers filtration (0.1–1.0 MPa/m). With such high values of the starting pressure gradient, filtration is possible only in the bottomhole zone, which does not correspond to field practice.

### Experimental data

In order to establish the regularities of filtration in low-permeability reservoirs, we set up special experiments to clarify the type of dependence of the filtration rate on the pressure gradient. We have studied a collection of terrigenous core samples with average porosity values (from 0.16 to 0.22) and low values of absolute permeability (from 0.7 to 6 mD). As the fluid

we have used natural oil from one of the oilfields of the Volgograd region, with a low viscosity ( $\sim 0.77$  mPa·s) and low gas content ( $\sim 101$  m<sup>3</sup>/t). The operating pressure gradient varied from 0.01 to 80 MPa/m.

In the course of the experiments, thermobaric conditions were simulated corresponding to the studied reservoirs. Pressure and temperature were selected based on the results of well testing.

Residual water saturation was created by the method of a semipermeable membrane on a capillarimeter, the value of which, based on the approved value of the initial oil saturation of 0.40 adopted in the geological and physical characteristics of the studied object, was 0.60. The creation of the initial oil saturation is carried out by additional saturation under vacuum of individual samples with kerosene, which is then replaced by oil when pumping the latter in an amount of 10 pore volumes before the start of the experiment. Gas did not participate in filtration.

Graphical dependences of the filtration rate on the pressure gradient in these experiments are shown in Fig. 1.

The studies have shown that most experimental plots demonstrate a non-linear relationship between velocity and pressure gradient. The absence of experimental points in the region of low filtration rates is explained by the difficulty of creating low pressure gradients ( $grad p < 0,05 \frac{\text{MPa}}{\text{m}}$ ). However, the form of the experimental dependences shows that with a decrease in the pressure gradient, the recorded filtration rate gradually tends to zero, and zero values of the filtration rate correspond to zero values of the pressure gradient.

The traditional approximation of the results of our experiments on filtration in low-permeability reservoirs by linear dependences demonstrates non-zero values of the velocity of 0.5–3.0 m/day at zero values of the pressure gradient. This determines the non-physical nature of such an approximation. As noted above, a linear approximation of the filtration law in low-permeability reservoirs leads to the appearance of a starting (initial) pressure gradient, which is not observed according to the data of field studies.

The studies carried out have shown that the dependence of the filtration rate on the pressure gradient, in accordance with the physics of the process, with a high degree of correlation is approximated by the following relation

$$V = \frac{A}{\mu} (grad p)^{\gamma} \quad (2)$$

It should be noted that at large values of the pressure gradient, the power-law dependence (2) with a good approximation tends to a linear dependence of the filtration rate on the pressure gradient, and the filtration law can be represented as a linear relationship with the initial pressure gradient. However, in a wide range

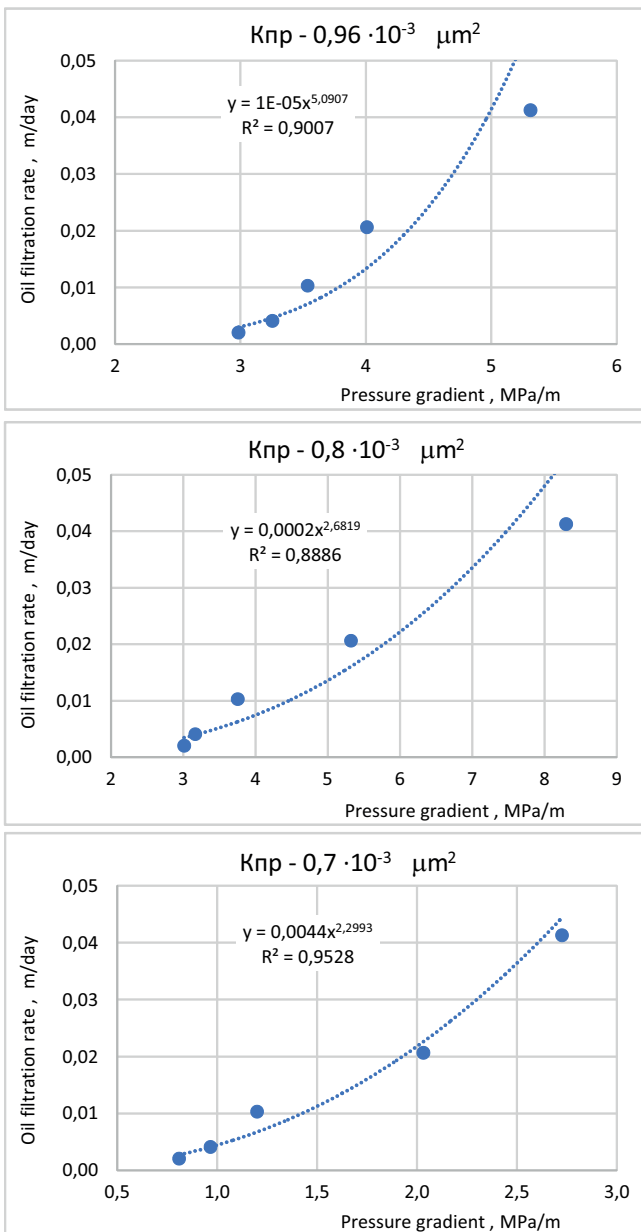


Fig. 1. Experimental dependences of the oil filtration rate on the pressure gradient for various samples

of pressure gradients, the filtration law with an initial pressure gradient is inapplicable. Thus, the nonlinear filtration law with an initial pressure gradient is a special case of a more general power law, but it describes only a high-gradient filtration region.

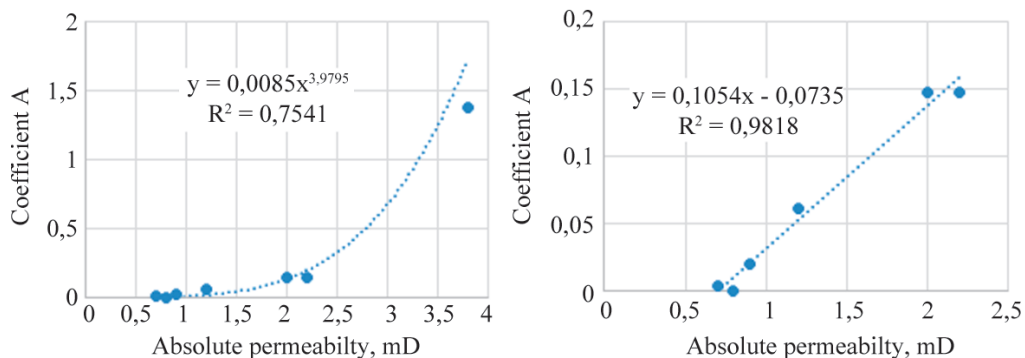


Fig. 2. Dependence of coefficient A on absolute permeability

In relation (2), the parameters  $A$  and  $\gamma$  are constant coefficients. The coefficient  $A$  at a constant viscosity of the filtering fluid is not analogous to the relative permeability, but can be related to the absolute permeability through the porosimetric characteristic of the reservoir.  $A$  coefficient dimension

$$A \left[ m^2 / \left( \frac{MPa}{m} \right)^{\gamma-1} \right]$$

Fig. 2 shows the dependence of the  $A$  coefficient on the values of the absolute permeability of the reservoir. Studies have shown that the dependences of the  $A$  coefficient on the values of the absolute permeability coefficients differ for samples with different microstructure (with different values of the absolute permeability).

The dependence is linear for samples with permeability values less than 3.8 mD; for samples with permeability less than 6 mD there is nonlinear dependence. For the values of the exponent  $\gamma$ , no dependence on the values of the absolute permeability was found. The experimentally established variation range of the nonlinearity parameter  $\gamma$  is from 1 to 5.

Since the concept of permeability is based on the linear Darcy's law, this parameter does not have a rigorous definition for nonlinear filtration. Having adopted a linear filtration law in a small range of pressure gradient changes and using the Darcy ratio in this range, it is possible to determine some "effective" relative permeability ( $k_f$ ), which will not be constant. This permeability will depend on the applied pressure gradient. Comparing the expressions for the filtration rate according to Darcy's law and according to the nonlinear power law, one can obtain an expression for the oil relative permeability ( $k_f$ ).

$$k_f = A(grad p)^{\gamma-1} \tag{2^1}$$

The relation of the oil relative permeability with pressure gradient is illustrated in Fig. 3.

As can be seen from the graphs, the pressure gradient, up to a certain critical value, has a strong effect on the oil relative permeability. When this value is exceeded, the permeability is practically independent of the pressure gradient. The region of insensitivity (self-similarity)

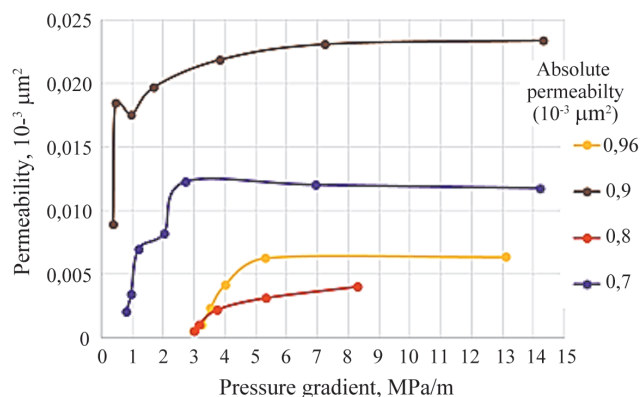


Fig. 3. Dependence of oil permeability ( $K$ ) on the pressure gradient

on the dependence of oil permeability on the pressure gradient corresponds to the range of pressure gradients, in which the filtration law is linear. Linearity determines the constancy of oil permeability in the specified range of the pressure gradient. This permeability can be characterized as “asymptotic”. No relationship was found between the values of the asymptotic relative permeability and the values of the absolute permeability.

### Physical interpretation of experimental data

The power law of filtration that we have established is fundamentally different from the previously established nonlinear laws. The nonlinearity of filtration differs from the previously known nonlinear laws in the mechanisms of nonlinearity. Forchheimer’s law is valid for high filtration rates and Reynolds numbers. The filtration law for viscoplastic fluids is determined by their non-Newtonian properties and demonstrates the initial pressure gradient (Basniev et al., 1993). A similar law has been established for low-permeability reservoirs (Baikov et al., 2013), but for them the initial pressure gradient exist due to the physicochemical interactions between the reservoir and the fluid (Li Xuanzhan, 2015; Baoquan, 2011; Hao, 2008; Wang, 2011). All these laws assume constant values of the permeability coefficient. The nonlinear law established by us demonstrates variable values of the fluid permeability coefficient, which leads to a change in the generally accepted filtration models (Lobkovsky et al., 2019). The power-law form of the nonlinear filtration law is explained by the specifics of the intrapore interaction of rocks and fluids (during filtration through pore systems complicated by the presence of boundary layers in them) and local pressure gradients arising during filtration.

In the boundary layer, the molecular interaction of fluids with the pore surface is quite strong, but the structure and stability of the layer depend not only on the physicochemical activity of the fluid – pore surface system, but also on the acting pressure gradient of the filtering fluid, which tends to deform this layer. In standard reservoirs, the formation of the boundary layer

is associated mainly with physicochemical interactions, while the influence of the pressure gradient is subordinate (Markhasin, 1977). The formation of a quasicrystalline layer leads to a decrease in the relative permeability, which is typical for standard reservoirs. The destruction and deformation of the layer under the influence of the pressure gradient, which is characteristic of low-permeability reservoirs, on the contrary, leads to an increase in the relative permeability. In low-permeability reservoirs, with an increase in the pressure gradient, weak intermolecular forces forming the boundary layers are overcome, and the filtering pore space increases. Due to the specifics of this increase, there arise the anomalies in the dependence of the filtration rate on the pressure gradient, which affect the relative permeability. An increase in relative permeability with an increase in the pressure gradient occurs until the boundary layers are completely deformed, then almost all of the open pore space will be involved in the filtration process. This is the specificity of the interaction of rocks and fluids during filtration in low-permeability reservoirs, which determines the nonlinearity of filtration and a special mechanism for the relative permeability formation.

### Nonlinear model of filtration in low-permeability reservoirs

The following nonlinear filtration model was proposed. Until a certain critical value of the pressure gradient  $G$  is reached, the change in permeability is modeled as a function of the pressure gradient  $\Omega(\text{grad } p)$ . When the gradient is exceeded, filtration proceeds according to a linear law.

First, consider the problem statement in general, i.e. without specifying a specific type of function. The change in the relative permeability ( $k_f$ ) under the influence of the pressure gradient is described by the expression (3)

$$k = k_f \Omega(\text{grad } p) \quad (3)$$

where  $k$  is the relative permeability at the current value of the pressure gradient.

In the general case, in addition to the above-described change in permeability under the influence of nonlinearity, we also take into account the change in permeability due to technogenic pollution (when drilling) and compressibility (when testing the formation) using special dimensionless functions for changing the permeability along the radius  $A(r)$  and change in permeability with pressure change  $f(p)$  (Zaitsev et al., 2004). The change in the natural permeability of the formation  $k_0$  as a result of technogenic pollution at the stage of formation drilling is modeled by the function  $A(r)$ , and the change in permeability due to deformation of the rock skeleton at the stage of inflow stimulation is modelled by the function  $f(p)$ , so that as a result of the action of these factors the technogenically altered oil

relative permeability will be determined by the following expression (Zaitsev et al., 2004):

$$k_f = k_0 A(r) f(p)$$

where  $k_f$  – oil relative permeability [ $\mu\text{m}^2$ ];  $k_0$  – unchanged reservoir permeability for oil [ $\mu\text{m}^2$ ];  $A(r)$  and  $f(p)$  are dimensionless functions of contamination and filtration compression.

Then the permeability, taking into account the nonlinearity of filtration, in accordance with relation (3), can be represented as follows:

$$k = k_0 f(p) A(r) \Omega(\text{grad} p)$$

for  $dp/dr < G$ . (4)

When the critical pressure gradient is exceeded, the permeability is constant and is expressed by the usual formula (3) (Zaitsev et al., 2004)

$$k = k_0 f(p) A(r)$$

for  $dp/dr > G$ . (5)

Let us consider radially symmetric filtration in a reservoir that is infinite in strike and thickness.

For the radially symmetric case, the filtration equations can be converted to polar coordinates as follows:

$$\frac{1}{r} \frac{d}{dr} (rk_0 f(p) A(r) A_0 \Omega(\text{grad} p)) = 0, \quad (6)$$

$$\frac{1}{r} \frac{d}{dr} (rk_0 f(p) A(r) \frac{dp}{dr}) = 0 \quad (7)$$

Thus, we obtain a system of differential equations to determine the pressure distribution. The boundary conditions will be the conditions at the well and the contour

$$p|_{r=r_k} = p_k, \quad p|_{r=r_c} = p_c \quad (8)$$

In addition, it is necessary to fulfill the condition of the gradient equality to the critical one at the border of the zones. Let us denote the radius when the critical pressure gradient is reached through  $r_l$ .

$$\left. \frac{dp}{dr} \right|_{r=r_l^-} = \left. \frac{dp}{dr} \right|_{r=r_l^+} = G \quad (9)$$

The solution to this system and the possibility of obtaining an analytical solution depend on the form of the function  $\Omega$  and accordingly the form of the resulting differential equation.

Lets consider the case revealed during the experiments – the power-law dependence of the permeability on the pressure gradient.

$$\Omega(\text{grad} p) = \left( B \frac{dp}{dr} \right)^\alpha \quad (10)$$

where  $\alpha$  is a dimensionless constant ( $\alpha \geq 0$ ), and  $B$  is a constant with the dimension [ $\text{m}/\text{MPa}$ ], which depends on the formation and fluid properties.

$$f(p)^{1/\alpha} \frac{dp}{dr} = \left( \frac{C}{rA(r)} \right)^{1/\alpha}$$

or

$$\begin{aligned} \Phi(p_c, p) &= C_1 \Psi(r_c, r) + C_2, \quad \text{for } r < r_1, \\ \Phi_\alpha(p_1, p) &= C_3 \Psi_\alpha(r_1, r) + C_4, \quad \text{for } r > r_1, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \Psi(r_i, r_j) &= \int_{r_i}^{r_j} \frac{dr}{(A(r)r)}, \quad \Phi(p_i, p_j) = \int_{p_i}^{p_j} f(p) dp \\ \Psi_\alpha(r_i, r_j) &= \int_{r_i}^{r_j} \frac{dr}{(A(r)r)^\alpha}, \quad \Phi_\alpha(p_i, p_j) = \int_{p_i}^{p_j} f(p)^{1/\alpha} dp \end{aligned} \quad (12)$$

The constants  $C_1$ – $C_4$  are determined from the boundary conditions (8), as well as from the condition of gradients and pressures equality at the zones boundary (9). The result of solving the system is the obtained values of the radii of the zones boundaries as well as the pressures for them.

Then the expression for the flow rate can be obtained in the following form

$$Q = 2\pi h \frac{k_0 \Phi(p_c, p_1)}{\mu \Psi(r_c, r_1)} \quad (13)$$

For a compressible reservoir, it is impossible to obtain analytical solutions of system (11)–(13), therefore, in this work, to analyze the influence of nonlinearity parameters on the well flow rate, we consider an incompressible reservoir, i.e.  $f(p) = 1$ . Then system (11–12) with boundary conditions (8–9) can be reduced to one equation with respect to the variable  $r_l$ .

$$\begin{aligned} p_k - p_c &= Gr_1 A(r_1) \Psi(r_c, r_1) + \\ &G^{(\alpha+1)} r_1 A(r_1) \Psi_\alpha(r_1, r), \end{aligned} \quad (14)$$

which greatly simplifies the solution of the original system, making it possible among other things to obtain analytical solutions. The following figure (Fig. 4) shows the change in reduced pressure versus radius on a logarithmic scale in the absence of contamination.

The dots mark the distance at which the critical pressure gradient  $G$  is reached. On curve 1 (at  $\alpha = 0$ ), as well as on the left side of the graph (where the pressure gradient is greater than the critical one), in the absence of contamination, the flow proceeds according to a linear law, which is expressed in straight sections of curves on the graph. With an increase in the values of the nonlinearity parameter  $\alpha$ , the curve deviates more and more from the classical straight line.

In the presence of contamination ( $A(r) \neq 1$ ) (Fig. 5), both parts of the curves become nonlinear.

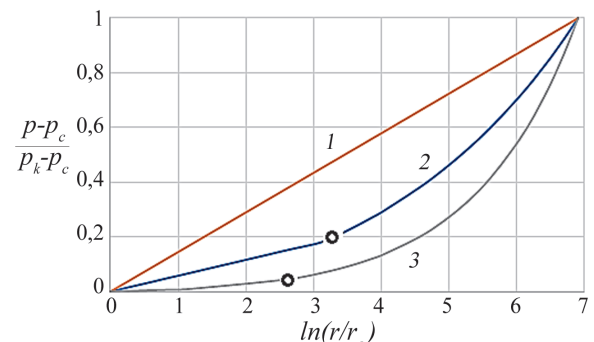


Fig. 4. Dependence of the reduced pressure in the near-wellbore zone on the logarithm of the reduced radius at the values of the nonlinearity parameter:  $\alpha = 0$  (1);  $\alpha = 0.5$  (2);  $\alpha = 2$  (3)

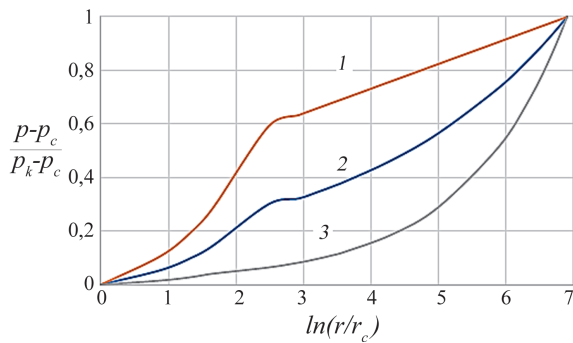


Fig. 5. Dependence of the reduced pressure in the near-wellbore zone on the logarithm of the reduced radius for the case of a contaminated near-wellbore zone with the values of the nonlinearity parameter:  $\alpha = 0$  (1);  $\alpha = 0.5$  (2);  $\alpha = 2$  (3)

As can be seen from Fig. 5, the values of the nonlinearity parameter  $\alpha$  in relation (10) have a decisive effect on the pressure distribution in the near-wellbore zone, and, consequently, on the well flow rate. At the same time, another important parameter characterizing nonlinear filtration is the radius  $r_1$ , at which the value of the critical gradient  $G$  is reached and the filtration mode is changed.

Let's consider the dependence of the radius  $r_1$  on the parameters characterizing the flow nonlinearity –  $\alpha$  and  $G/G_0$ , where  $G_0$  is the pressure gradient in the well in the case of inflow without taking into account the nonlinearity zone:

$$G_0 = \frac{p_k - p_c}{r_c A(r_c)} \frac{1}{\Psi(r_c, r_k)}$$

As can be seen from the graph (Fig. 6), starting from some  $\alpha$ , the critical radius becomes equal to the radius of the well, i.e. linear mode is not achieved. At small values of the critical gradient, the critical radius ( $r_1$ ) is reached far from the well, at the same time, starting from some values of  $G$ , the implementation of the linear mode is impossible.

### Influence of nonlinearity factors on well flow rate during well testing

The influence of the nonlinearity parameters, in accordance with expression (13), on the well production rate is manifested through the specifics of the pressure distribution in the near-wellbore zone. Fig. 7 shows the

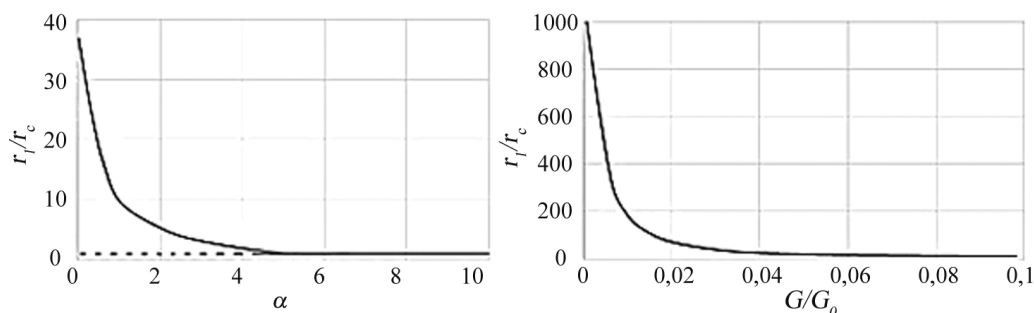


Fig. 6. Dependence of the radius of reaching the critical pressure gradient on the nonlinearity parameter  $\alpha$  and the critical gradient

dependence curves of the relative production rate ( $Q/Q_0$ ).  $Q_0$  is a well flow rate without taking into account the dependence of permeability on the pressure gradient, i.e. for  $\alpha = 0$ .

As can be seen from the graphs (Fig. 7), an increase in the values of the nonlinearity parameter  $\alpha$  causes a decrease in the production rate in the entire considered range of drawdown values per formation, while the effect at low drawdowns is higher. As can be seen from the curves, with a sufficiently large coefficient  $\alpha$  (curve 3), before the creation of a depression sufficient to achieve a critical pressure gradient, there is practically no flow during well testing, and with a depression close to  $p_k$ , a drop in production rate as a result of the nonlinearity factors can reach 90 %.

In the case of small values of  $G/G_0$ , the critical gradient is achieved in any situation, the influence of the coefficient  $\alpha$  on the flow rate is minimal. The magnitude of the critical depression depending on the  $G/G_0$  and  $\alpha$  parameters can reach  $0.35p_k$ , which causes a very strict requirements for testing conditions for fixing inflow from low-permeability reservoirs.

The proposed model also allows to calculate the minimum required pressure drop to create an inflow. With an increase in  $\alpha$ , the minimum required pressure drop to create an inflow also increases. Another factor in the growth of the minimum required drawdown is the near-wellbore zone contamination during the formation

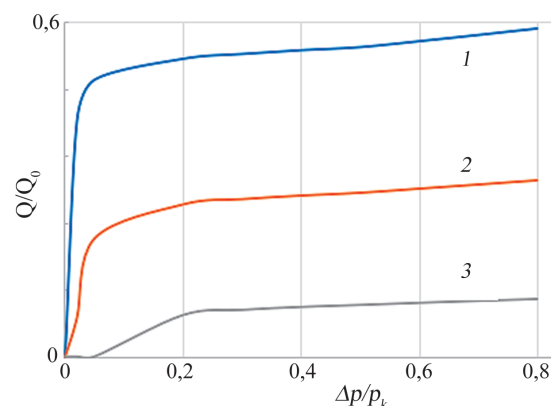


Fig. 7. Dependences of the well flow rate during well testing on the drawdown value ( $\Delta p$ ) related to the pressure on the circuit ( $p_k$ ) at various values of the nonlinearity parameters:  $\alpha = 0.5$  (1);  $\alpha = 1$  (2);  $\alpha = 2$  (3);  $G/G_0 = 0.06$

drilling. Contamination intensifies the nonlinearity effects during low-permeability formations well testing.

### Conclusion

The authors carried out the analysis of the geological and physical reasons for nonlinear filtration in low-permeability reservoirs.

It was found that for low-permeability reservoirs there is no constant coefficient of oil relative permeability.

The target oil permeability is a function of the pressure gradient and in the region of small gradients is described by a nonlinear dependence on this parameter.

A nonlinear filtration model is proposed to assess the efficiency of well testing.

The influence of nonlinearity factors on the well production rate has been established.

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