

# Particle removal by two-phase filtration flow from a porous medium under wave action

A.I. Nikiforov, R.V. Sadovnikov\*

*Institute of Mechanics and Engineering, FRC Kazan Scientific Center of the Russian Academy of Sciences, Kazan, Russian Federation*

**Abstract.** The paper discusses the influence of wave action on the process of detachment and removal of particles from a porous body by a two-phase filtration flow. When modeling this process, the problem of the influence of the wave field on the force under the action of which the particles are detached from the pore walls is solved. For the first time, a pore-size distribution function is used for its solution. An expression for the critical flow velocity under wave action has been obtained. Critical frequency value of wave action depends on the capillary radius and the smaller the capillary radius is, the higher frequency is needed to enhance the effect of the action. At higher frequency of oscillation the peak of maximum change in the thickness of the sedimentary layer is shifted towards the pores of small radius. To maintain the influence of the wave field on the filtration parameters of the porous medium, the wave action should be carried out at a dynamically changing frequency range to increase the coverage of the effect of as many pores as possible. It is shown that particle removal during wave action increases due to the action of inertial forces, which reduce the influence of forces holding the particles on the pore surface.

**Keywords:** wave action, porous medium, two-phase filtration, control volume method

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## Introduction

The wave action is used in practice as one of the effective methods of stimulation in order to improve its filtration properties. The wave field intensifies particle detachment from the walls of pore channels and their removal by the two-phase filtration flow out of the porous body. As a consequence, the transparency, porosity and permeability of the porous medium are improved, i.e. there are structural changes in its filtration properties. To describe this phenomenon, it is necessary to apply methods of micromechanics with subsequent transfer of processes that take place in each of the pores to the entire porous medium

In the case of single-phase flow in porous medium mathematical model (Shekhtman, 1961), in which kinetic relations for description of the processes of particles detachment from the pore walls (suffosion) and deposition of particles on the capillary walls (colmatation) with blocking of some pore channels are proposed. Similar kinetic relations were used in (Gruesbeck et al., 1982).

In the work (Sharma et al., 1987) a mathematical model for the general class of problems related to the transport of particle suspension in porous media was formulated. A porous medium is represented by a network of pore channels with narrowings. Particle retention and permeability reduction are written in terms of different mechanisms of particle sedimentation and detachment. The sedimentation and detachment velocities are estimated using appropriate physical models. The concept of an effective medium is applied to determine the distribution of fluid flow in the network and to calculate permeability. The representation of the porous medium by a network of channels in combination with balance relations for particle sedimentation and detachment velocities provides a consistent model that has applications in filtration and migration processes.

Yu.I. Kapranov proposed an approach to describe the motion of monodisperse suspension in a porous medium (Kapranov, 1989, 1999, 2000), which allowed the author to form an accurate system of equations for the integral characteristics and give an opportunity to describe changes in the structural parameters of the medium in the process of colmatation. The class of equilibrium regimes, distinguished in this case, has served as the basis for the construction of determining relations for integral models and the study of changes

\*Corresponding author: Roman V. Sadovnikov  
E-mail: [sadovnikov@imm.knc.ru](mailto:sadovnikov@imm.knc.ru)

in the medium and flow parameters in conditions of equilibrium regimes.

In (Nikiforov et al., 1998), a mathematical model of dispersed particles removal by a two-phase flow in an oil reservoir, in which the porous medium is represented as two interpenetrating continua, one of which is associated with moving fluids and particles, and the other with non-moving ones, is proposed. An equation is obtained which defines the dynamics of the pore size distribution function. The velocity of change in the pore channel radius and the velocity of decrease in the number of capillaries of a certain radius are estimated based on the model representation of the porous medium as a bundle of capillaries with narrowings. Appropriate expressions for dynamic porosity, permeability and mass transfer between two media are written.

The paper (Nikiforov et al., 2013) presents a mathematical model of dispersed particle removal by a two-phase filtration flow, which takes into account the processes of particle sedimentation in the porous body and their detachment from the pore walls. It was shown that the bulk of the solid rock is transferred out of the oil reservoir by the water flow. The effect of redistribution of solid particles in pore channels of different sizes was detected.

The wave processes in porous fluid-saturated media, wave acceleration mechanisms of filtration processes are considered in the monograph (Ganiev et al., 2008), as well as methods of creating additional filtration flows and the possibilities of controlling wave processes in the reservoir are investigated. On the basis of theoretical and experimental studies of forced nonlinear oscillations of fluid-saturated porous media the effects that appear in the wave fields and affect filtration processes in productive formations, as well as the cleaning of bottomhole zones are studied.

Hydro- and thermodynamic phenomena that arise in porous media saturated with oil, gas and (or) water during propagation in them of elastic waves with oscillation frequencies varying from infrasonic to ultrasonic are studied in (Kuznetsov et al., 2001). The physical basis of vibration and acoustic stimulation to improve oil and gas recovery and increase the rate of development of depleted and flooded oil fields are described. The issues of post-displacement of oil, after the impact of vibrations are considered. The effect of oscillations on oil post-displacement from clay-bearing reservoirs, which include the terrigenous reservoirs containing up to 15–20 % of clay minerals as a cementing material, was investigated using different types of models of porous media and cores. It was found that in clay-bearing reservoirs under the influence of oscillations the microflows and location of active clay minerals are changed in relation to the skeleton of the reservoir. According to the experiments on the packed

reservoir models, it was found that the oscillations are changing the character of the influence of mineralization of injected water on oil recovery.

In (Selyakov et al., 1995), the percolation model is used to analyze the possible mechanisms of irreversible changes in the permeability of a saturated porous medium under acoustic influence. Among the mechanisms of interaction of the acoustic field with the saturated porous medium, leading to irreversible changes in permeability, such mechanisms are highlighted and analyzed as: dissipation of energy of the viscous Poiseuille flow in the presence of mutual movement of liquid and solid phases, destruction of the surface layer of pore channels under the action of tangential stresses arising at the border of solid and viscous fluids, cavitation in pore channels during acoustic wave propagation, dissipation of acoustic field energy by the appearance of a leaky fluid. Based on estimates and calculations, the authors have established that for acoustic impact with the most applicable frequency  $\sim 10^4$  Hz, the underlying mechanism leading to changes in the filtration characteristics of the medium is a mechanism associated with the dissipation of acoustic field energy due to the appearance of a fluxless fluid motion, the so-called thermal slip.

The localization of energy release in separate groups of capillaries leads to destruction and transport of cementing substance (clay, biotite, etc.) from them as a result of thermoelastic stresses and, consequently, increase in the radii of pore channels. Since permeability  $\sim r^4$ , where  $r$  is capillary radius, then even its small change is sufficient to change its filtration characteristics and, consequently, effective permeability of the whole medium.

In this work the phenomenon of removal of dispersed particles from a porous body by a two-phase filtration flow under wave action is investigated. When modeling this process, the problem of particles detaching from the wall of the pore channel by a two-phase fluid flow under the action of the wave field is solved. For the first time, a pore-size distribution function is used to solve it. The arising Cauchy problem for the pore-size distribution function in each grid node of the computational domain and the filtration problem as a whole are solved by the method of control volumes. It is shown that particle removal under wave action increases due to the action of inertial forces, which reduce the influence of forces holding the particles on the pore surface.

### Mathematical model

The mathematical model of particle transport by two-phase filtration flow is based on the laws of conservation of mass and momentum, which are completed by closing relations. The closing relations are derived using representation of porous medium as a bundle

of capillaries, which allowed to describe the process of particle detachment at microlevel and its impact on porous medium as a whole at macrolevel. The basic hypotheses and assumptions, derivation of equations and relations are detailed in the works, a review of which is given in (Gazizov et al., 2002; Nikiforov et al., 2011; Nikiforov et al., 2013). In this article, the main resulting equations and relations are reported. In the large-scale approximation, the model is represented by the following phase and component mass conservation equations (Nikiforov et al., 2013):

$$\frac{\partial}{\partial t}(mS_i) + \nabla \mathbf{v}_i = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(C_i m S_i) + \nabla(C_i \mathbf{v}_i) = q_{Ci}, \tag{2}$$

where  $i$  – denotes the phase ( $i = o$  – oil,  $i = w$  – water),  $m = m(x, y, z)$  – dynamic porosity,  $\mathbf{v}_i$  – phase filtration velocity,  $S_i$  – saturation of porous medium with phase,  $C_i$  – volume concentration of particles in the phase,  $q_{Ci}$  – intensity of particles volume change in the phase. Equations of motion of phases are written in the form of generalized Darcy’s law

$$\mathbf{v}_i = -k \frac{k_{ri}(S_i)}{\mu_i} \nabla p_i, \tag{3}$$

where  $p_i$  – pressure in the phase;  $\mu_i$  – dynamic viscosity of the phase;  $k_{ri}$  – relative phase permeability,  $k$  – absolute permeability of the porous medium. The condition of complete saturation of porous body with phases has the form:

$$S_o + S_w = 1. \tag{4}$$

The system of equations (1)–(4) describes the dynamics of particle transport by phases (oil and water) and should be supplemented with boundary and initial conditions when solving a specific problem. To describe the structural changes in porosity and absolute permeability of reservoir due to the detachment of particles from the walls of pore channels, methods of micromechanics of porous media are applied. In this case porous medium is represented as a bundle of capillaries of different cross-section.

### Equation for the pore size distribution function

The pore space structure (dynamic porosity and absolute permeability) is changed when particles are detached from the walls of the pore channels, so it is necessary to supplement the system with closure relations, which are constructed using the pore size distribution function for an idealized model of a porous medium in the form of a bundle of capillaries of different radii and formulated as a boundary Cauchy problem. The Cauchy problem for the pore – size distribution function has the form (Gazizov et al., 2002; Nikiforov et al., 2011):

$$\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial r}(u_r \varphi) = 0, \tag{5}$$

$$\varphi(r, 0) = \varphi^0(0), \tag{6}$$

where  $u_r$  – the rate of expansion of pore channels due to the suffosion process.

### Expansion of the pore channel

Particles can be detached by fluid flow from pore channel wall when fluid flow reaches a certain critical velocity, at which the condition of equilibrium of forces acting on a particle is violated. On one side the hydrodynamic pressure force of liquid flow acts on a particle tending to involve the particle into the motion, and on the other side the particle is held on the pore channel wall by gravitation forces, friction and surface roughness. Supposing the laminar flow of liquid, the hydrodynamic force of liquid pressure on the particle coincides with the Stokes force at average velocity of the incoming flow equal to average velocity of liquid in the capillary of circumferential cross-section  $r_0 < r \leq r_0 + l$ , where  $l$  – particle diameter,  $r_0$  – capillary radius. The holding force is proportional to the weight of the particle (the analogue of the frictional force with some friction coefficient  $C_f$ ). The equilibrium condition for the forces acting on the particle is written in the form (Nikiforov et al., 2013):

$$6\pi\mu_i u_{mi} \frac{l}{2} \left[ 1 - \left( 1 - \frac{l}{2r_0} \right)^2 \right] - C_f \frac{\pi}{6} l^3 \rho_p g = 0, \tag{7}$$

where  $C_f$  – fictitious resistance coefficient of the particle in the phase,  $\rho_p$  – density of the particle. Hence, the critical velocity at which the particle is detached from the wall is determined by the formula:

$$u_i^* = C_f \frac{l^2 \rho_p g}{18\mu_i \left[ 1 - \left( 1 - \frac{l}{2r_0} \right)^2 \right]}. \tag{8}$$

The intensity of suffosion is proportional to the difference between the average velocity of liquid flow in the capillary and the critical velocity (Shekhtman, 1961)

$$u_r = \frac{\partial h}{\partial t} = -\delta(u_m - u^*)h \frac{r + 0.5h}{r}.$$

Under the conditions of two-phase filtration, the intensity of suffosion is assumed to be

$$u_{ri} = -\delta(u_{mi} - u_i^*)h \frac{r + 0.5h}{r}, \tag{9}$$

where

$$u_{mi} = \frac{|\mathbf{v}|r^2}{8\zeta k(k_{ro}/\mu_o + k_{rw}/\mu_w)\mu_i}, \mathbf{v} = \mathbf{v}_o + \mathbf{v}_w, \tag{10}$$

$u_{ri}$  – rate of pore channel expansion for the phase,  $u_{mi}$  – average phase flow velocity in the capillary,  $\zeta$  – tortuosity of the capillary,  $\delta$  – kinetic constant of suffosion,  $r$  – radius of the capillary,  $\mathbf{v}$  – total velocity of filtration,  $h$  – the thickness of the sediment layer of particles in the capillary.

If  $q_{Ci}$  is the intensity of change in the volume of

particles in the  $i$ -th phase, then in time  $\Delta t$  the volume of particles will change by the value of  $q_{Ci}\Delta t$ . During the same time the capillary radii will change by the value of  $\Delta r = u_{ri}\Delta t$ , which will lead to decrease of porosity and transparency. Considering that the pore volume occupied by the  $i$ -th phase is proportional to the saturation  $S_p$ , we obtain

$$q_{Ci}\Delta t = mS_i \left[ \int_0^\infty (r + \Delta r)^2 \varphi dr - \int_0^\infty r^2 \varphi dr \right] / \int_0^\infty r^2 \varphi dr$$

or, neglecting by the term of the second order of smallness, we obtain

$$q_{Ci} = 2mS_i \int_0^\infty ru_{ri}\varphi dr / \int_0^\infty r^2 \varphi dr. \quad (11)$$

Since, due to the chaotic distribution of pore channels, it is not possible to specify which pores the oil moves through, and which water, therefore, in general, in the elementary volume of the porous medium the capillary expansion rate is taken as a sum (Nikiforov et al., 2013):

$$u_r = u_{ro} + u_{rw}. \quad (12)$$

The equilibrium condition for the forces (7) acting on the particle during the wave action is supplemented by the inertial force arising due to vibrations of the wall of the capillary:

$$6\pi\mu_i u_{mi} \frac{l}{2} \left[ 1 - \left( 1 - l/(2r_0) \right)^2 \right] - C_{fi} \frac{\pi}{6} l^3 \rho_p g + C_{fi} \frac{\pi}{6} l^3 \rho_p \omega^2 r = 0. \quad (13)$$

Then the expression for the critical flow velocity under wave action will have the following form:

$$u_i^* = \begin{cases} C_{fi} \frac{l^2 \rho_p (g - \omega^2 r)}{18\mu_i \left[ 1 - \left( 1 - l/(2r_0) \right)^2 \right]}, & \text{if } \omega \leq \sqrt{g/r}, \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

i.e. the critical flow velocity for those capillaries in which the condition  $\omega \leq \sqrt{g/r}$  is violated, is considered to be equal to zero and all particles can enter the flow, as the filtration velocity at a given pressure drop is not equal to zero. Analysis of formula (14) allows to conclude that increasing the frequency of vibration with wave action leads to a decrease in the value of the critical flow velocity, at which the particles can be detached from the capillary wall by the flow, in contrast to the critical flow velocity without wave action. Thus, the force of inertia of the capillary wall, which occurs during wave action, reduces the effect of forces holding the particle to the capillary wall. The limiting frequency value  $\omega^* = \sqrt{g/r}$  depends on the pore channel (capillary) radius and the smaller the channel radius, the higher frequencies are needed to enhance the effect of the action. To maintain the effect of the wave field on the critical flow velocity, the wave action should be carried out on a dynamically changing frequency range to increase the coverage of the action on as many capillaries of different radii as possible. As particles are detached from pore channel

walls, the particles are transferred out by the fluid, thus increasing the pore channel radius and consequently increasing its flow capacity, i.e. increasing the porosity and permeability of the porous medium as a whole. To account for these changes, it is necessary to write down additional relations.

**Changes in the pore space structure.** The description of the change in the pore space structure is based on the representation of the porous medium as a bundle of parallel capillaries (Nikiforov et al., 2013). The change of dynamic porosity is defined through the transparency and has the following form:

$$m = m_f m_0, \quad m_f = \int_0^\infty r^2 \varphi dr / \int_0^\infty r^2 \varphi_0 dr, \quad (15)$$

where  $m_f$  is a factor of porosity increase,  $m_0$  is an initial porosity value. The formula for the change in absolute permeability is obtained by comparing the expression for the fluid flow through the bundle of capillaries with the expression for the fluid flow according to Poiseuille's law and has the form:

$$k = k_j k_0, \quad k_j = \int_0^\infty r^4 \varphi dr / \int_0^\infty r^4 \varphi_0 dr, \quad (16)$$

where  $k_j$  is a factor of increasing the permeability of the porous medium,  $k_0$  is an initial value of permeability.

Analysis of formulas for porosity and absolute permeability factors of the medium, shows that they are proportional to  $r^2$  and  $r^4$ , respectively, where  $r$  is the radius of the capillary. This allows us to conclude that even a small change in capillary radius is enough to change its filtration characteristics and, consequently, effective porosity and permeability of the whole medium. Similar conclusion was made by the authors (Selyakov et al., 1995) when analyzing effective permeability for percolation model of porous medium.

The unknowns in the mathematical model (1)–(16) are pressure  $p_p$ , saturations of pore space with oil and water  $S_p$ , concentrations of particles  $C_p$ , velocities of filtration of phases  $v_i$ .

**Solution method.** Numerical approximation of equations (1)–(16) on space is constructed by method of control volumes. The searched fields of pressure, water saturation and concentration of particles are related with nodes of rectangular grid and with middle of cell height, and the cell itself serves as a control volume. Integration of the equations over the grid blocks (control volumes) is performed under the assumption that the values of formation characteristics are constant over the block. When calculating surface integrals in the equations for water saturation and the impurity concentration, the direction of flows is taken into account. To approximate time derivatives, the IMPES method scheme (Aziz et al., 1979) is used, which leads to a system of difference equations with respect to pressure  $p_p$ , water saturation of pore space  $S_w$ , particle concentration  $C_i$ .

The Cauchy problems for the pore size distribution functions (5)–(6), (9)–(11) are solved by the finite element method on a grid of one-dimensional finite elements using a linear representation of the desired functions on each element. An implicit scheme is used for time approximation. When integrating the convective terms, the characteristic directions are taken into account. The values of the pore-size distribution functions are calculated in the region for which this value has physical meaning, i.e. the mesh covered the finite part of the interval  $[0, \infty)$ , outside of which the solution did not propagate during the solution of problem (1)–(16). This limit is the maximum pore size of a pure (without sedimented particles) porous body (Nikiforov et al., 2011).

The method for solving the problem is described in detail in (Gazizov et al., 2002; Nikiforov et al., 2011), and its main steps are briefly described here. At each time layer, it is necessary to calculate: 1) pressure field in all nodes of the spatial grid, 2) water saturation field, 3) particle concentration fields, 4) total filtration velocity, 5) pore size distribution function, 6) structural changes in porosity and permeability of the formation.

The method of numerical solution of the problem is rather complicated due to the fact that to calculate the fields of pressure, saturation, concentration of impurities, it is necessary to calculate the structural changes in porosity and permeability at each time step in each node of the grid of the calculation area. This involves solving Cauchy problems for the particle size distribution functions and requires significant computer time and resources. The computational acceleration issues in such problems were discussed in detail in (Nikiforov et al, 2017) for a computational cluster with distributed memory, and in (Nikiforov et al, 2019) for a computational system with a hybrid architecture, which significantly improved computational performance.

**Numerical results**

Numerical results were obtained for a sample of oil-saturated porous medium with linear dimensions  $(L \times B \times H) = (0.3 \text{ m} \times 0.1 \text{ m} \times 0.1 \text{ m})$  (Fig. 1). The following parameter values were taken:  $\rho_p = 2000 \text{ kg/m}^3$  is the density of particles,  $l = 0.000004 \text{ m}$  is the diameter of particles,  $\delta = 0.000005$ ,  $\zeta = 1$ ,  $\rho_o = 817 \text{ kg/m}^3$  is the density of oil,  $\mu_o = 5 \cdot 10^{-3} \text{ mPa} \cdot \text{s}$  is oil viscosity,  $\rho_w = 1182 \text{ kg/m}^3$  is water density,  $\mu_w = 1 \cdot 10^{-3} \text{ mPa} \cdot \text{s}$  is water viscosity,  $C_{fo} = 0.03$ ,  $C_{fw} = 0.3$  is a resistance coefficients

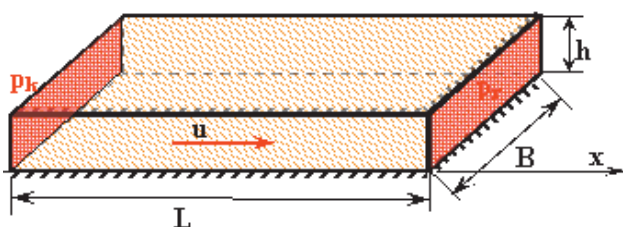


Fig. 1. Sample of a porous medium

of spherical particles in oil and water, respectively;  $m_0 = 0.2$ ,  $k_0 = 0.6 \mu\text{m}^2$  – initial values of dynamic porosity and permeability of the porous medium. The initial size distribution of the pore radii is shown in Fig. 2.

Water enters through the entrance section  $x = 0$ , and liquid is withdrawn through the exit section  $x = L$ , which moves in the sample under the action of a constant pressure drop  $\Delta p = P_1 - P_2$ , where  $P_1$  – inlet pressure to the sample,  $P_2$  – outlet pressure. Capillary forces are neglected. Initial concentration of particles in phases is zero, i.e. all particles are on pore walls. Concentration of particles in water injected in the inlet section is equal to zero, i.e.  $C_w(0, t) = 0$ . Since only water enters through the inlet section and the residual oil is immobile, the boundary condition for the concentration of particles in the oil is assumed  $C_o(0, t) = 0$ . The top and the bottom boundaries of the sample  $z = 0$  and  $z = H$  are considered impermeable. Relative phase permeabilities were approximated by quadratic saturation functions  $k_{ro} = A_o(1 - S_o^* - S_w)^2$  and  $k_{rw} = A_w(S_w - S_w^*)^2$ , where  $A_o = 1$ ,  $A_w = 1$ ,  $S_o^* = 0$ ,  $S_w^* = 0$  is a residual oil saturation and connected water saturation of the reservoir, respectively. The grid of the computational domain has  $61 \times 11 \times 1 = 671$  cells. A grid of one-dimensional finite elements for calculating the pore size distribution function in each cell of the computational domain consisted of 26 finite elements.

The distribution of particle concentrations in water along the length of the sample is shown in Fig. 3 for pressure drop  $\Delta p = 0.01 \text{ MPa}$ , and in Fig. 4 for the pressure drop  $\Delta p = 0.002 \text{ MPa}$  for different moments of time:  $t_1 = 0.01$ ,  $t_2 = 0.03$ ,  $t_3 = 0.05$ ,  $t_4 = 0.056$ ,  $t_5 = 0.06$ ,  $t_6 = 0.1$ ,  $t_7 = 0.3$ ,  $t_8 = 0.6$ ,  $t_9 = 1$  day (dashed line – without wave action, solid line – with wave action). It can be seen that the smaller the pressure drop and, consequently, the filtration velocity, the greater the influence of wave action on the particle detachment.

In Fig. 5 at the same moments of time  $t_1, \dots, t_9$  the distribution of particle concentrations in oil along the length of the sample for the pressure drop  $\Delta p = 0.002 \text{ MPa}$

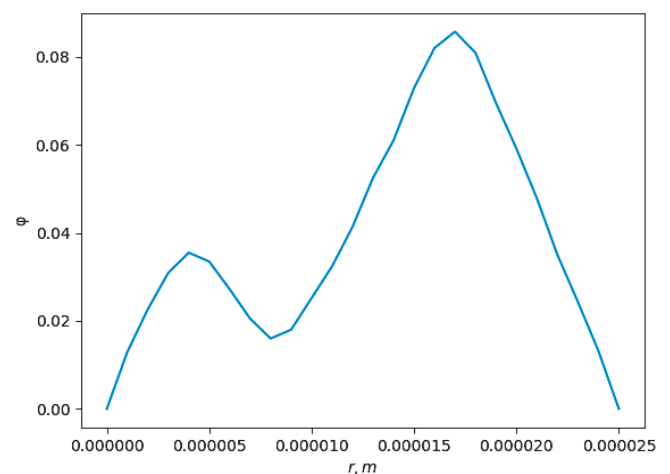


Fig. 2. Distribution of pore radii by size

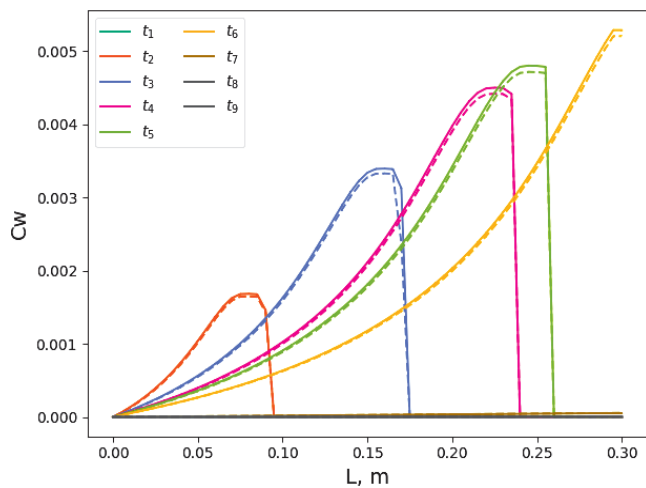


Fig. 3. Concentration of particles in water  $\Delta p = 0.01$  MPa,  $\omega = 2150$  Hz

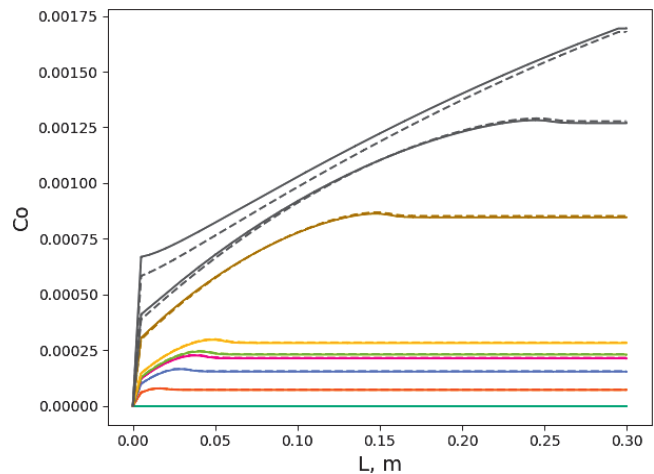


Fig. 5. Concentration of particles in oil,  $\Delta p = 0.002$  MPa,  $\omega = 2150$  Hz

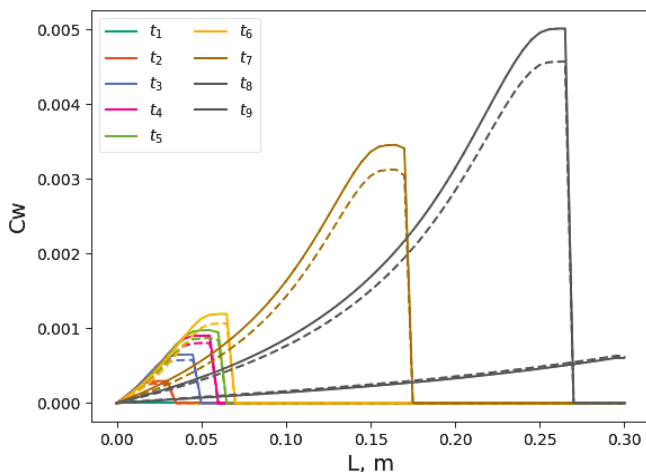


Fig. 4. Concentration of particles in water,  $\Delta p = 0.002$  MPa,  $\omega = 2150$  Hz

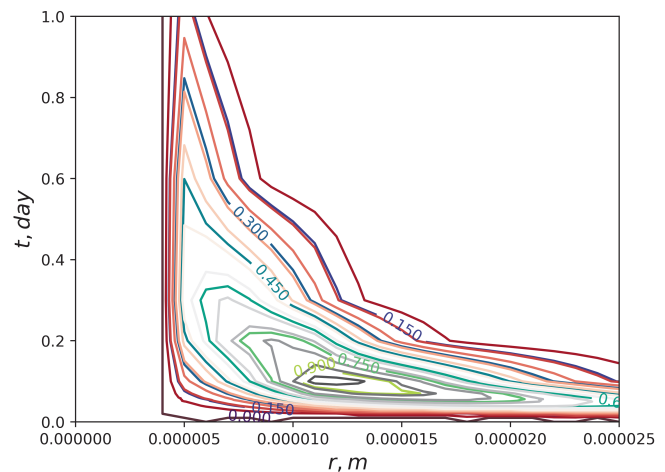


Fig. 6. Change of the thickness of the sedimentary particle layer ( $\omega = 150$  Hz)

is presented. It can be seen that during wave action, the concentration of particles detached from the capillary walls is higher in water than in oil, which is related to the values of friction coefficients of particles in liquids.

Fig. 6, 7 show in a dimensionless form the change in the thickness of the sedimentary layer  $\Delta h^* = \Delta h / (\Delta h_{\max} - \Delta h_{\min})$ , where  $\Delta h$  is the difference of thicknesses of sedimentary layer depending on the pore radii at different moments of time in the sample without wave action (i.e., removal due to the action of hydrodynamic pressure force only) and with wave action ( $\omega = 150$  Hz and  $\omega = 2150$  Hz),  $\Delta h_{\max}$ ,  $\Delta h_{\min}$  maximum and minimum values of thickness change over the entire range of pore channel radii and over all time moments, respectively. From the results presented in Fig. 6, 7 it can be seen that wave action increases the particle transport, the thickness of the sedimentary layer of particles in the pore channels decreases and, consequently, the filtration parameters of the porous medium: porosity and permeability increases. At higher frequency of oscillations the peak of maximum change of sedimentary layer thickness shifts towards small radius of pores (Fig. 7).

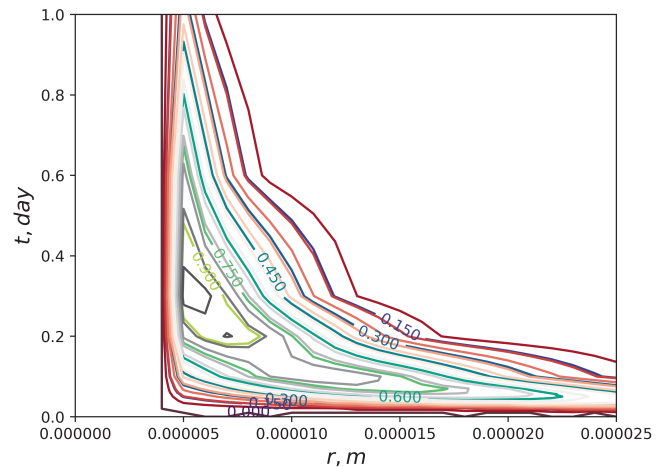


Fig. 7. Change of the thickness of the sedimentary particle layer ( $\omega = 2150$  Hz)

The changes of dynamic porosity and permeability for time moments  $t_1, \dots, t_9$  are shown in Fig. 8, 9 (dashed line – without wave action, solid line – with wave action, respectively). Thus, the presented mathematical model allows taking into account the influence of wave field on particle transport out of porous medium.

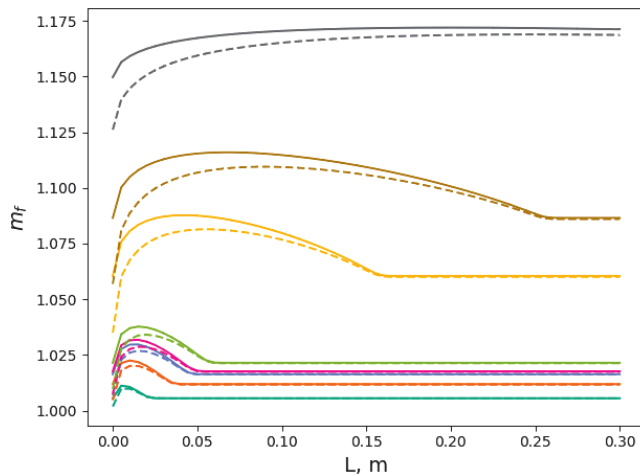


Fig. 8. Change of dynamic porosity of the sample,  $\Delta p = 0.002$  MPa,  $\omega = 2150$  Hz

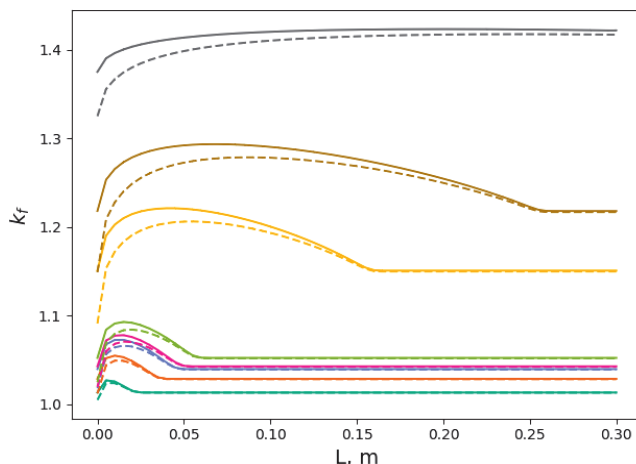


Fig. 9. Change of dynamic permeability of the sample,  $\Delta p = 0.002$  MPa,  $\omega = 2150$  Hz

**Conclusion**

A mathematical model of wave action on a porous medium at two-phase filtration to remove dispersed particles suspended on the pore walls has been developed. The expression for the critical flow velocity, which takes into account the influence of the wave field, is obtained. It is shown that the transport of particles during wave action increases due to the force of inertia, which reduces the effect of the force holding the particles on the pore surface. Critical value of the frequency of wave action depends on the radius of the capillary, and the smaller the radius of the capillary, the higher frequencies are needed to enhance the effect of the action. At higher frequency of oscillation the peak of maximum change in thickness of sedimentation layer is shifted towards the pores of small radius. The model does not take into account the processes of sedimentation of particles on the walls and pore blocking, which is planned in further research. The results can be used to simulate the processes of bottomhole zone treatment under vibration-wave action.

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**About the Authors**

Anatoly I. Nikiforov – DSc (Physics and Mathematics), Chief Researcher, Laboratory of Mathematical modelling of filtration processes, Institute of Mechanics and Engineering, FRC Kazan Scientific Center of the Russian Academy of Sciences

2/31, Lobachevsky st., Kazan, 420111, Russian Federation

Roman V. Sadovnikov – PhD (Engineering), Senior Researcher, Laboratory of Mathematical modelling of filtration processes, Institute of Mechanics and Engineering, FRC Kazan Scientific Center of the Russian Academy of Sciences

2/31, Lobachevsky st., Kazan, 420111, Russian Federation

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