Tensor Representation of Capillary Model of a Porous Medium (Theory and Experiment)

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Abstract. Generalization is given for the case of anisotropic porous media represented by a simple model of capillary porous medium. It is an idealized model in which the porous medium is a periodic lattice formed by mutually perpendicular cylindrical capillaries. It is assumed that each capillary system is characterized by its parameter d_a and stacking period a_a (a = 1, 2, 3). It is shown that for anisotropic porous media, the functions of pore density distribution by radius and the values of the equivalent pore diameter depend on the direction and are set by symmetric tensor of the second rank. Scalar values of equivalent pore diameter are calculated, as is customary in crystal physics, in the form of the tensor properties along a predetermined direction. In the article an idea is given of the permeability coefficients tensor for simple capillary model of porous media and showed that the direction permeability value is determined by the tensor composition of luminal factor and pore distribution density by radii. The proposed theoretical representations of pore distribution density by radii; equivalent pore diameter and permeability coefficients are tested on the experimental data obtained in the laboratory experiment on a real core material. The main directions of the permeability tensor are determined from the extreme values of the transmission velocity of ultrasonic waves through the lateral surface of the core. Measurements on the control sample confirmed the tensor nature of the permeability. Pore distribution curves by radii are obtained by tomographic studies of core samples (device SkyScan 1172). A good agreement between theoretical and experimental results is obtained.

Keywords: simple capillary model, anisotropic media, filtration properties, characteristic linear dimensions, luminal tensors, pore distribution density by radii.
DOI: 10.18599/grs.18.3.4

For citation: Maksimov V.M., Dmitriev N.M., Kuz'michev A.N. Tensor Representation of Capillary Model of a Porous Medium (theory and experiment). *Georesursy = Georesources*. 2016. V. 18. No. 3. Part 1. Pp. 166-170. DOI: 10.18599/grs.18.3.4

The generalization of classical filtering theory models to the case of anisotropic filtration properties is relevant, since the real hydrocarbon reservoirs are porous and fractured media, almost always have anisotropy. Increasing the information content of the anisotropic core study for oil and gas field is also an important practical problem for determining the structure of pore space, filtration properties of the reservoir and estimation of reserves, as well as for optimum placement of wells and direction of horizontal wells.

1. **A simple capillary model for anisotropic porous media**. For a description of seepage flows in porous media mathematical models are often used, in which the real porous medium is represented in the form of capillary systems, cracks, packages of balls, etc. The most popular models include a simple capillary model of direct parallel capillaries whose radii are distributed according to a F(r) distribution function, so that

$$F(r) = \int_0^r f(r)dr, \qquad (1.1)$$

where f(r) is the density distribution of pore radius, f(r)dr – fraction of the pore space with capillaries whose radii lie in the interval from r to r + dr (Dmitriev, 1995; Dmitriev, Maksimov, 2001; Dmitriev et al, 2012.). In this approach, knowing f(r) we can determine the effective radius and capillary permeability. The effective capillary radius is determined by the formula (Romm, 1985)

$$r_э = \int_0^\infty rf(r)dr, \qquad (1.2)$$

to determine the permeability we have the formula

$$k = \frac{m}{8}\int_0^\infty r^2 f(r)dr. \qquad (1.3)$$

Equations (1.1) – (1.3) are valid only for the extremely anisotropic media: the model of rigid tubes, which allows for the filtration only along one direction. In this case, the porosity equals to luminal factor (Dmitriev, 1995; Dmitriev, Maksimov, 2001), and the permeability is represented by the equality $k = mr_э^2/8$. If we consider the three-dimensional capillary model of anisotropic porous medium, it is clear that the equality (1.1) – (1.3) can be written for each of the main directions of the tensor permeability coefficients in

$$k_\alpha = \frac{s_\alpha}{8}\int_0^\infty r^2 f_\alpha(r)dr, \; r_i^э = \int_0^\infty rf_i(r)dr,$$

$$F_i(r) = \int_0^r f_i(r)dr, \alpha, i = 1,2,3 \qquad (1.4)$$

where s_α – are principal values of the luminal factors tensor (Dmitriev, Maksimov, 2001), hereinafter by repeated Greek indices summation is not performed; the summation is only over repeated Latin indices.

Introduction along the main lines of luminal si values and density functions of pore distribution by radius $f_i(r)$ in fact means postulating of luminal tensors s_i and distribution density of capillaries radius $f_{ij}(r)$. This approach allows moving from the scalar notation, such as (1.2) and (1.3), to tensor and determining tensor of the effective radii of the capillaries in the form

$$r_{ij}^{э} = \int_0^\infty r f_{ij}(r) dr, \qquad (1.5)$$

and for the tensor of permeability coefficients in a simple capillary model we write equation

$$k_{ij} = \frac{1}{8}\int_0^\infty r^2 s_{ik} f_{kj}(r) dr \qquad (1.6)$$

As noted above, over repeated Latin indices in (1.6) and so on summation is meant.

Values of the effective pore diameter and the permeability coefficient along an arbitrary direction is defined as the tensor property in a given direction (Dmitriev et al., 2012) by

$$r^{э}(n) = r_{ij}^{э} n_i n_j = \int_0^\infty r f_{ij}(r) n_i n_j dr, \qquad (1.7)$$

$$k(n) = k_{ij} n_i n_j = \frac{1}{8}\int_0^\infty r^2 s_{ik} f_{kj}(r) n_i n_j dr, \qquad (1.8)$$

respectively. For the main directions the formulas (1.7) and (1.8) give the equation (1.4).

For orthotropic symmetry of filtration properties, or in the main frame of reference, which obviously coincides with the principal axes of the tensor of absolute permeability, tensors s_{ij} and $f_{ij}(r)$ have the form

$$s_{ij} = s_1 e_i^1 e_j^1 + s_2 e_i^2 e_j^2 + s_3 e_i^3 e_j^3, \qquad (1.9)$$

$$f_{ij}(r) = f_1 e_i^1 e_j^1 + f_2 e_i^2 e_j^2 + f_3 e_i^3 e_j^3, \qquad (1.10)$$

where s_α and f_α are the principal values of tensor s_{ij} and $f_{ij}(r)$ respectively; e_i^α – components of the unit vectors directed along the principal directions of the tensor, $e_i^\alpha e_j^\alpha$ – dyad, α=1,2,3. Substituting expressions (1.9) and (1.10) in (1.7) and (1.8) gives the following formulas

$$r^{э}(n) = r_{ij}^{э} n_i n_j = \int_0^\infty r(f_1 n_1^2 + f_2 n_2^2 + f_3 n_3^2) dr, \qquad (1.11)$$

$$k(n) = k_{ij} n_i n_j = \frac{1}{8}\int_0^\infty r^2 \left(s_1 f_1 n_1^2 + s_2 f_2 n_2^2 + s_3 f_3 n_3^2\right) dr \qquad (1.12)$$

At present, the complex laboratory experimental studies of reservoir properties were held, taking into account the anisotropy of the formation, the reservoir of hydrocarbons (Dmitriev et al, 2012;. 2014; Kuzmichev 2013). The latter allows testing theoretical constructs on the results of laboratory tests.

2. **Experimental verification of generalization of a simple capillary model for anisotropic porous media**. To conduct laboratory experimental research we selected cylindrical core of laminated cemented sandstone with height and diameter of 100 mm, which was extracted and dried.

It was believed that the layering of sandstone is perpendicular to the axis of cylinder symmetry, so one of the main directions of permeability coefficients tensor were supposed to be known. To determine the main directions in the plane of layering we used instrument "Uzor 2000" (Dmitriev et al, 2012;. Kuzmichev, 2013). The instrument measured the velocity of ultrasonic waves through the side surface of the core in km/s in increments of 30° (Fig. 1).

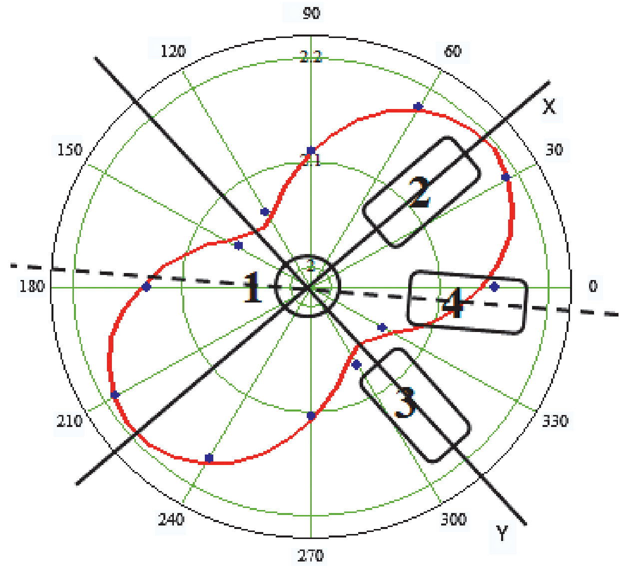

Fig. 1. The velocity profile of ultrasonic waves in the core section; the location of the cut core samples of smaller diameter.

The main directions of symmetric second-rank tensors defining material properties are consistent with extreme values of ultrasonic wave velocity (Dmitriev et al., 2014). According to the measurements we identified the main directions of the tensor permeability coefficients in the layering plane. Next, from the core source four smaller samples were sawn with 25 mm diameter and 30 mm in length. Three cores (samples 1 to 3) were sawn along the principal directions. The fourth sample is check on the bisector of the angle between the extreme directions.

The latter sample is used to test the hypothesis that the core symmetry axis coincides with the main direction of the permeability tensor and that the resulting values are indeed tensor components in the layering plane (Fig. 1). The figure shows the profile of the ultrasound for one section of the core. During sonic test of core it was noted that the main line changes slightly from one section to another over the entire height of the core. However, the maximum deviation does not exceed 10°, that, within the measurement error.

Further, porosity and absolute permeability were determined for all oriented samples in filtration of helium at atmospheric conditions. Permeability measurements were performed on a lab-tested scientific center of analytical and special core analysis of SC "VNIIneft". Measurement results are shown in Table 1. The repeated permeability measurements showed little change in the third decimal place.

Control sample, as noted above, is used to check tensor nature of characteristics: permeability, pore distribution density by radii. Let us check tensor nature of resulting permeability values. Using resulting values and k_2 we can calculate permeability value for all directions $k(\vec{n}) = k_{ij} n_i n_j$, find theoretical value k_4 and compare it with experimental value. Substitution of the numerical values in the formula for directed permeability gives k_4=634·10^{-15} m^2. Comparison of the theoretical and experimental values shows that the difference is less than 2%.

To obtain distribution curves of pore by radius we used CT scanner SkyScan 1172. Tomography scanner allows obtaining two-dimensional slices and three-dimensional models of pore space with high resolution (the resolution limit of 1 micron). In slices, by calculation we can obtain pore distribution density function by radii for each of the chosen direction. 1200 slices were made for each sample. The results are shown in Fig. 2-5.

According to the formula (1.7) we can calculate the effective radius of capillaries $r_\alpha^э$ for all directions and then from the equation $k_\alpha = s_\alpha (r_\alpha^э)^2 / 8$ we can determine the principal values of the luminal tensor sa. As a result, the following values were obtained of $r_\alpha^э$ и s_α: $r_1^э$=5,0 micron, $r_2^э$=4,63 micron, $r_3^э$=4,69 micron, $r_4^э$=4,88 micron, s_1=22,1%, s_2=21,6%, s_3=24,3%, s_4=21,7%. According to formulas of directed values $r^э(n)=r_{ij}^э n_i n_j$ and $s(n)=s_{ij} n_i n_j$ we can compare the theoretical and experimental values of the tensor of the effective radii and luminal factor. Substitution of the numerical values in the formula gives the theoretical values of the effective radius of 4.82 micron and luminal factor 21.85%.

Comparison of the theoretical and experimental values shows that the difference is less than 2%.

A similar check is allowed by the pore distribution density function by radii. Comparison of the theoretical density function values of the pore by radius with experimental values for the control sample is shown in Fig. 5.

Sample	Porosity, %	Permeability by gas, mD ($10^{-15} m^2$)
«Z» (1)	18,64	(k_3) 668
«X» (2)	18,83	(k_1) 689
«Y» (3)	18,74	(k_2) 579
«Control» - 45° (4)	18,54	(k_4) 644

Table 1. The results of measuring the porosity and permeability on all core samples.

The latest series of studies was to determine the residual water saturation for the same oriented sample.

To convert laboratory parameters Hassler-Bruner method was chosen (Kuznetsov et al., 2010;. Mikhailov, 2008), allowing with a high degree of accuracy to determine the magnitude of capillary pressure at the outboard end of the sample, to estimate the corresponding saturation and build dependencies on capillary pressure and saturation. Results of the study are shown in Fig. 6 for samples in the direction of

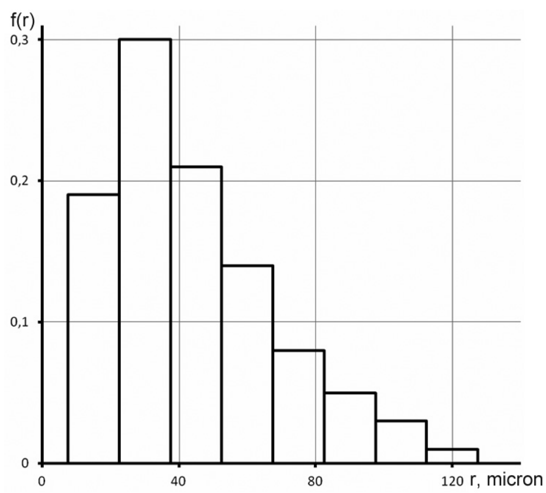

Fig. 2. Histogram of pore distribution density function by radii for the sample oriented along the x-axis.

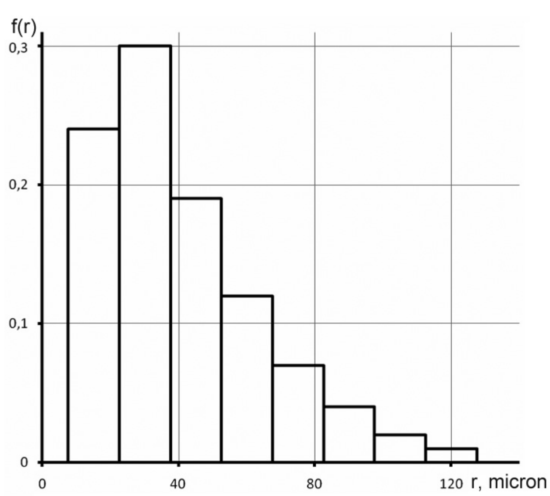

Fig. 3. Histogram of pore distribution density function by radii for the sample oriented along the y-axis.

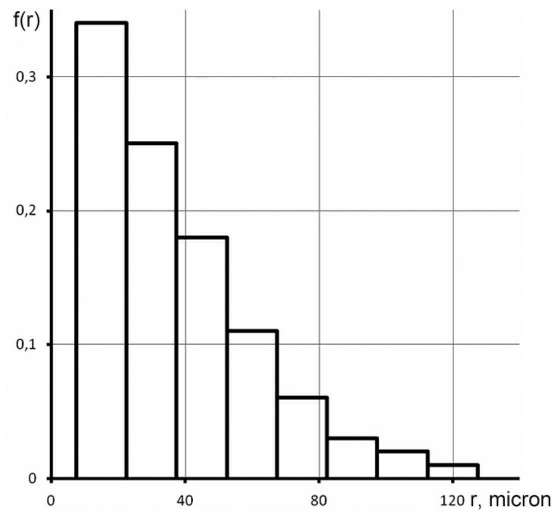

Fig. 4. Histogram of pore distribution density function by radii for the sample oriented along the z-axis.

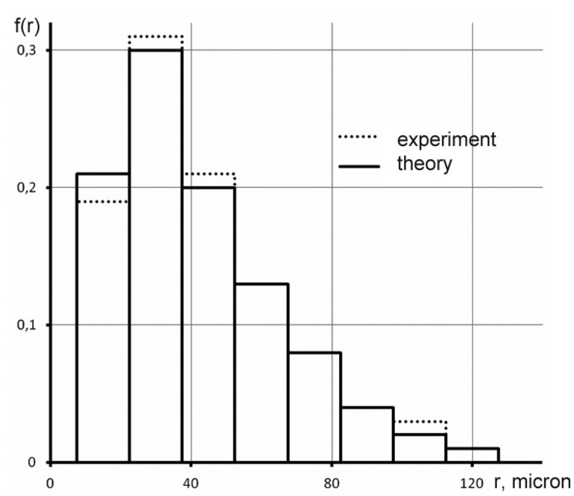

Fig. 5. Histogram of pore distribution density function by radii for the control sample and comparison of theoretical and experimental histograms.

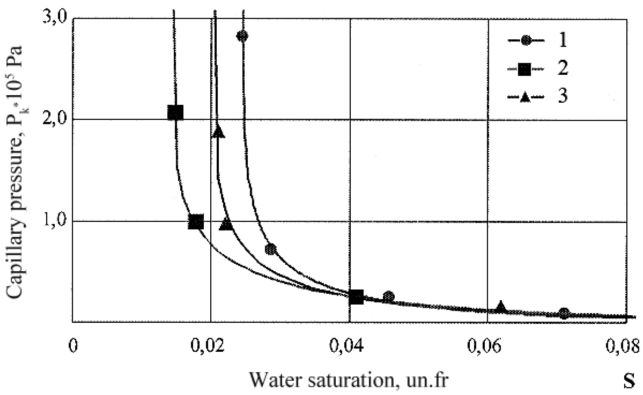

Fig. 6. Dependence of the capillary pressure from water saturation for the X, Y directions and control sample. 1 – capillary pressure curve with the experimental points for the sample X; 2 – capillary pressure curve with the experimental points for the sample Y; 3 – capillary pressure curve with the experimental points for the control sample.

X and Y in the layering plane and for control sample 4. The comparison results of theoretical and experimental values of capillary pressure on the control sample suggest tensor nature of the capillary pressure in anisotropic media.

An important consequence of this study is the effect of capillary pressure depending on the direction of flow. This implies that the capillary pressure is not a universal function of saturation for the anisotropic rock sample, but depends on the direction of flow, of the direction of stimulation. Confirmation of this fact requires further experimental and theoretical studies to identify the physical mechanisms of this effect, the structural characteristics of the pore space, the physical characteristics and physico-chemical interaction of 'fluid-rock' system, the nature of the layer wetting and other factors. The experiments should be repeatable for various rock samples at different scales of heterogeneity.

Let us note that the dependence of the relative phase permeabilities from the flow direction can be regarded as an established fact, confirmed by a number of experiments for clastic and carbonate samples with transversely isotropic, orthotropic and monoclinic symmetry of filtration properties; the tensor nature of the relative phase permeability is confirmed; their analytical dependence from saturation and structural parameters in different directions is obtained; a quantitative assessment is given for the contribution of the anisotropy effects in the performance with 'anisotropic' relative phase permeability included in a hydrodynamic model (Dmitriev et al., 2012;. 2014; Ter-Sarkisov et al., 2012, and a number of other publications).

Conclusion

A generalization is made for presentation of simple capillary model of a porous medium in the case of porous media with anisotropic filtration properties. It is shown that for anisotropic porous media values of equivalent pore diameter, pore distribution, luminal factor and density function by radii depend on the direction and are set by symmetric tensor of the second rank. The results of the theoretical constructs are confirmed by laboratory experimental studies on the core.

The work is methodological in nature. The main objective was to establish the equivalence of the tensor representation of the capillary model to "geometrical" characteristics (porosity, luminal factor, pore distribution function by size) of anisotropic porous medium followed by applying an integrated research method of the core. Further development of this method is associated with the study of core samples of deep horizons (6-8 km).

Financing

The work is performed in the framework of Programm 1,4P of the Presidium of the Russian Academy of Sciences (sub-program "Oil from the deep horizons of sedimentary basins – a source for replenishment of the hydrocarbon resource base; theoretical and applied aspects").

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Manuscript received June 9, 2016