LOCAL REFINEMENT OF THE SUPER ELEMENT MODEL OF OIL RESERVOIR

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Abstract. In this paper, we propose a two-stage method for petroleum reservoir simulation. The method uses two models with different degrees of detailing to describe hydrodynamic processes of different space-time scales. At the first stage, the global dynamics of the energy state of the deposit and reserves is modeled (characteristic scale of such changes is km/year). The two-phase flow equations in the model of global dynamics operate with smooth averaged pressure and saturation fields, and they are solved numerically on a large computational grid of super elements with a characteristic cell size of 200-500 m. The tensor coefficients of the super element model are calculated using special procedures of upscaling of absolute and relative phase permeabilities. At the second stage, a local refinement of the super element model is constructed for calculating small-scale processes (with a scale of m/day), which take place, for example, during various geological and technical measures aimed at increasing the oil recovery of a reservoir. Then we solve the two-phase flow problem in the selected area of the measure exposure on a detailed three-dimensional grid, which resolves the geological structure of the reservoir, and with a time step sufficient for describing fast-flowing processes. The initial and boundary conditions of the local problem are formulated on the basis of the super element solution. This approach allows us to reduce the computational costs in order to solve the problems of designing and monitoring the oil reservoir.

To demonstrate the proposed approach, we give an example of the two-stage modeling of the development of a layered reservoir with a local refinement of the model during the isolation of a water-saturated high-permeability interlayer. We show a good compliance between the locally refined solution of the super element model in the area of measure exposure and the results of numerical modeling of the whole history of reservoir development on a detailed grid.

Keywords: super elements method, numerical simulation, petroleum reservoir, local refinement, reservoir treatments simulation, two phase flow, downscaling

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The development of geological and technical measures of exposure on oil reservoirs and an increase in the overall share of hard-to-recover reserves raise the requirements for numerical modeling of flooding of oil deposits. The arising problems require both the detailedness of the solutions obtained and the increase in the computational speed when carrying out adaptation or optimization calculations. At the same time, the dimensionality of the grids detailing the fine geological structure of the entire reservoir is so great that their application for mass flow calculations leads to excessive computational costs.

To reduce the dimension of the calculated grids, averaging techniques are usually used (rescaling, upscaling) (Durlofsky, 1998; Panfilov, Panfilova, 1996; Belyaev, 2004; Mazo, Potashev, 2017 (a); Mazo, Potashev, 2017 (b)). At the same time, the possibility of describing small-scale flow processes characteristic for complex geological and technical measures is lost. An alternative option for accelerating computation is to use detailed grids not simultaneously for the entire deposit, but in local subareas. Examples of this approach are the family of multiscale methods (Aarnes et al., 2004, Arbogast, 2000; Efendiev et al., 2006; Gautier et al., 1999; Jenny et al., 2006; Pergament et al., 2010), which come to the construction of a detailed velocity field based on the solution of the pressure equation on a coarse grid and the subsequent solution of the saturation transfer equation on a fine grid. In this case, a significant part of the computational work for calculating the saturation is unnecessary for modeling the global dynamics of water flooding.

This paper demonstrates the feasibility of applying a two-stage super element modeling, using detail models of varying degrees for describing different-scale processes. Such an approach, in our opinion, allows us not only to reduce computational costs in order to solve the problems of oil reservoir design, but also to improve the accuracy of calculations in comparison with many conventional methods.

The choice of the method for local refinement of the super element model depends on the exposure method during the geological and technical measures. When

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modeling area measures, such as polymer flooding, the effect of which is occurred in the extended areas of interaction between injection and production wells, the method of fixed stream tubes (Mazo et al., 2017) can be used. This approach allows us to use high-resolution computational grids due to the decomposition of the three-dimensional problem into a series of two-dimensional ones. In this paper, we consider a method for local refinement of the model by isolating a small section of the geological and technical measures and constructing a small three-dimensional grid on it for a detailed calculation of the short-term consequences of isolating the watered perforation interval of a single production well.

To calculate the global development dynamics, a super element model is being built (Mazo, Bulygin, 2011; Mazo et al., 2013; Bulygin et al., 2013; Mazo et al., 2015) on large (200-500 m horizontally and 10-100 m vertically) unstructured computational grids with a number of cells comparable to the number of wells in the field. This makes it possible to reduce the computation time by hundreds of times in comparison with the calculations traditionally using grids with a pitch of 30-50 m. Satisfactory accuracy of calculations is achieved by formulating problems for smooth averaged fields of pressure and saturation, as well as performing upscaling of reservoir properties (Mazo, Potashev, 2017 (a); Mazo, Potashev, 2016) and the modified functions of the relative phase permeabilities (Mazo, Potashev, 2017 (b); Potashev, Abdrashitova, 2017). The super element approach allows us to carry out calculation of design parameters of oil reservoir development and predict the dynamics of the energy state of the deposit. At the same time, the super element method does not allow the modeling of relatively fast small-scale processes, for example, accompanying geological and technical measures to enhance oil recovery.

To describe such processes, a solution built on a super element grid must be locally refined in the area of the geological and technical measures. On the selected section of the reservoir, the two-phase flow problem is solved on a detailed spatial grid that resolves the geological structure of the reservoir, and with a time pitch sufficient to describe fast processes. The principal question of modeling the development of a single section is the formulation of the initial and boundary conditions on the basis of the super element solution. These conditions define a one-way connection between the global and local refined solutions.

To demonstrate the proposed approach, an example of a two-stage super element modeling of the development of a small oil deposit of a layered structure, penetrated by a series system of vertical perfect wells, is given (Figures 1, 2). The presence of a highly permeable interlayer leads to water breakthrough to production

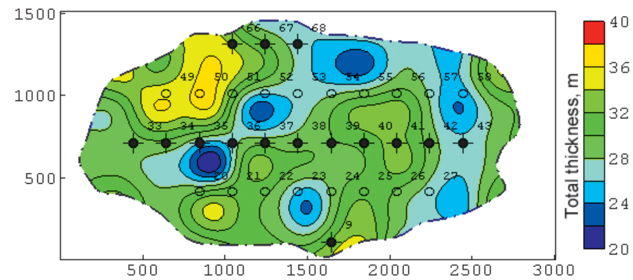

Figure 1. Projection to the horizontal plane of a model oil reservoir with cutoff along the outer contour of the oil content [1]

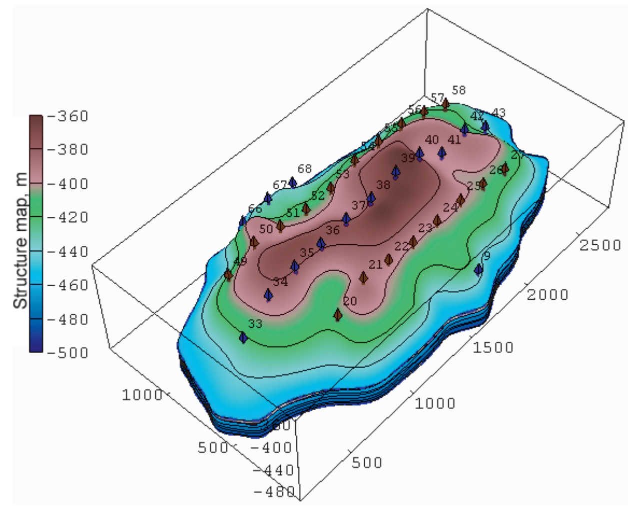

Figure 2. Three-dimensional representation of a model oil reservoir with a cut-off along the outer contour of the oil content

wells. To isolate the water inflow, a partial pouring of the perforation interval on a separate well is performed. This leads to a sharp change in the filtrational flows in the vicinity of the conducted geological and technical measures. Simulation of the consequences of this measure is carried out with the help of local refinement of the super element solution.

1. Local refinement method of a numerical solution

Let us describe the general sequence of actions for two-stage super element modeling.

1. The whole region of the reservoir D is covered by a super element grid. Absolute permeability and relative phase permeabilities are upscaled. Numerical modeling of the global dynamics of reservoir development is performed on the time interval $0 \le t \le T$. Averaged grid pressure functions $P(\mathbf{x},\ t)$ and water saturation $S(\mathbf{x},\ t)$ are sought in $\mathbf{x} = (x, y, z) \in D,\ t \in [0, T]$.

2. The site $\Omega \subset D$ is given, which is composed of a small number of super elements; a short time interval $t \in [t_0, t_0 + \tau],\ \tau << T$ is given, at which it is required to perform a local refinement of the super element solution.

[1] To visualize the geological and flow models, special software was used (Mazo et al., 2012; Mardanov, Bulygin, 2012; Bulygin, Mardanov, 2017)

The boundary $\partial\Omega$ of the section Ω consists of the outer Γ and the inner γ parts. The outer part Γ is a continuous surface, which is the union of all the outer faces of the constituent parts of super elements. The inner part γ is represented by the set of surfaces γ_i of all wells located inside the section Ω.

3. The area of the section Ω is covered by a detailed computational grid for solving the two-phase flow equations for small-scale pressure p and saturation s in neglect of fluid compressibility, as well as capillary and gravitational forces (Barenblatt, 1984):

$$\beta\frac{\partial p}{\partial t}+\nabla\cdot\mathbf{u}=0,\quad \frac{\partial m\, s}{\partial t}+\nabla\cdot\left(f\,\mathbf{u}\right)=0,\quad \mathbf{u}=-\sigma\nabla p,$$

$$\sigma=\frac{k}{\mu}=\frac{k}{\mu_w}\varphi(s),\quad \varphi(s)=k_w(s)+\mathrm{K}_\mu k_o(s),\ \mathrm{K}_\mu=\frac{\mu_w}{\mu_o},$$

$$f(s)=k_w(s)/\varphi(s),\ k_w(s)=\mathrm{s}^n,\quad k_o(s)=(1-s)^n,$$

$$n=1\div 4. \qquad (1)$$

Here β – compressibility of the reservoir; $\mathbf{u}$ – total flow rate; f – water fraction in the total flow; σ – hydraulic conductivity; φ – mobility of the mixture; μ – dynamic viscosity of a two-phase fluid, μ_w, μ_o – viscosity of the water and oil phases.

4. To solve the task (1), the initial

$$t=t_0,\ \mathbf{x}\in\Omega:\quad p=p^0(\mathbf{x}),\ s=s^0(\mathbf{x}) \qquad (2)$$

and boundary conditions are given. On the outer boundary Γ a third-kind condition is set (Potashev et al., 2016)

$$\mathbf{x}\in\Gamma:\ \sigma\frac{\partial p}{\partial n}=-\alpha\left(p-P_e\right),\ \alpha=\frac{\sigma}{h} \qquad (3)$$

where P_e – is a superelement solution at a distance h from the boundary Γ in the direction of the outer normal $\mathbf{n}$ at time t_0. In the "input" areas Γ^{in}: $\mathbf{u}\cdot\mathbf{n}<0$ saturation s_Γ is additionally given, which is built on the large-scale saturation S. On the inner boundary of γ – surfaces of γ_i wells – nonlocal boundary conditions are set: the flow rates at constant pressure in the well are given

$$\mathbf{x}\in\gamma_i:\ -\int_{\gamma_i}\sigma\frac{\partial p}{\partial n}d\gamma=q_i(t),\ p=p_i=\text{const} \qquad (4)$$

On the surfaces of the injection wells, the condition $s = 1$ is additionally specified.

The initial distribution of saturation s^0 in (2) is constructed on a detailed grid by means of the procedure of de-scaling (downscaling) of the mean field S in super elements at time $t = t_0$. The function p^0 in the initial condition (2) is given as the solution of the stationary problem for the pressure p. Since $\tau << T$, functions P_e and s_Γ in the boundary conditions can be assumed to be time-independent.

5. The task (1) with conditions (2)-(4) of local refinement of the model on a detailed grid is solved. The grid functions $p(\mathbf{x}, t)$, $s(\mathbf{x}, t)$; $\mathbf{x}\in\Omega,\ t\in\left[t_0,t_0+\tau\right]$. are constructed.

6. The found pressure p and saturation s functions are used to calculate the technological performance of the wells at the section of geological and technical measures.

The second stage of super element modeling is the local refinement of the model (actions 2-6) – can be performed for an arbitrary number of sections of the reservoir and at arbitrary instants of time. At the same time, the first stage – the modeling of global development dynamics on the super element grid (action 1) – is performed only once and does not involve the use of small-scale fields p, s for some refinement of the functions P, S.

2. An example of two-stage modeling

Let us consider an example of local refinement of the super element water flooding model. The geological characteristics of the reservoir, location of wells, and their operating modes were generated specifically to illustrate the proposed methodology. The geological model of the deposit is formed by three interlayers and two weakly permeable bridges. The permeability of the second interlayer was set much higher than the permeability of the first and third (Table 1). The average length of the deposit in two orthogonal directions was 2.5 km and 1.5 km (Figures 1, 2). The map of the total thickness of the reservoir is shown in Figure 1; parameters of the geological model are shown in Tables 1, 2. Absolute permeability of interlayers was calculated through porosity according to the Kozeny equation (Kozeny, 1927; Daigle, Dugan, 2009; Yang, Aplin, 2007).

Characteristic \ Interlayer	1	Bridge	2	Bridge	3
Average porosity, un. fr.	0.16	0.05	0.35	0.05	0.22
Average permeability, 10^{-15} m^2	6	0.15	120	0.15	20

Table 1. Average characteristics of interlayers

Characteristic	Minimum value	Average value	Maximum value	Standard deviation	Variability, m
Total thickness, m	20	30	40	4	300
Thickness of interlayer, m	0	7.5	15	8	300
Relative porosity within the interlayer, un. fr.	0.8	1	1.2	0.3	100

Table 2. Statistical parameters of the characteristics distribution of the geological model

At the beginning of development (01.2000), the reservoir is completely saturated with oil. The viscosities of water and oil were set equal to $\mu_w = 0.001$ Pa s, $\mu_o = 0.002$ Pa s. The functions of the relative phase permeability (1) were specified in the form of quadratic dependences (n = 2).

The reservoir is developed by a series system of 33 wells (Figure 1), 15 of which are injection wells with a constant injectivity of 150 m^3/day, and 18 – production wells with a constant flow rate of 100 m^3/day. The average well grid spacing is 200 m. All wells are vertical and initially perfect in the degree of reservoir penetration. On the production well 52 at the time 01.01.2004 the perforation interval is poured in the area of highly permeable interlayer in order to isolate the water inflow (Figure 3). This geological and technical measure is selected as the reason for the local refinement of the super element model.

On the super element grid, the whole reservoir development was modeled since 01.2000. The grid contained 90 super elements with an average diameter of 225 m (Figure 4). The height of the super elements coincided with the full height of the reservoir, that is, each super element contained all the interlayers and bridges.

For local refinement, a section around well 52 consisting of 18 super elements and containing 8 surrounding wells was determined (Figure 4). This section was covered with a detailed computational grid with an average cell diameter of 15 m, vertically the reservoir was divided into 5 finite volumes according to the structural surfaces of the interlayers and bridges. The constructed detailed computational grid contained 17900 finite volumes. The solution of tasks (1)-(4) on the detailed grid was built on a time interval covering the moment of the geological and technical measure under consideration – from 07.2003 to 01.2006. To specify the initial and boundary conditions of the local model, a super element solution – the pressure P and the saturation S – was used at the time 07.2003.

In constructing the initial saturation distribution s^0, a generalization to the three-dimensional case of the simplest technique was used as the procedure of downscaling (Stiles, 1949; Dykstra, Parsons, 1950; Bulygin, 1974). Two assumptions are used in it: (1) more penetrable interlayers are flooded in the first place; 2) the displacement of oil by water is of a piston type, therefore part of the interlayers is completely watered, and the other part is saturated with oil. The distribution of a given volume of water within the reservoir reduces to finding such an absolute permeability value k* that layers with a permeability $k < k^*$ are saturated with oil, and layers with $k > k^*$ are saturated with water. With reference to the considered three-dimensional task, the generalization of this technique was realized as follows:

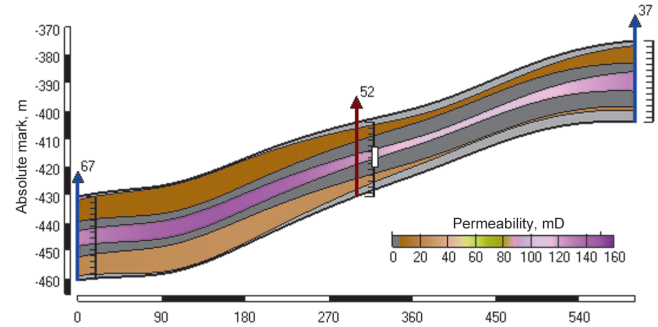

Figure 3. Arrangement of perforation intervals (solid lines) and pouring (white color)

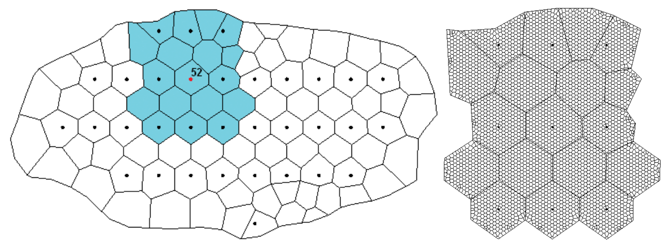

Figure 4. Covering the area of the deposit with a super element grid, the region of local refinement of the model (on the left) and its coverage with a detailed grid (on the right)

1) for each super element $V \subset \Omega$ with average porosity M and average saturation S known at the moment of downscaling, the elements V_i of the detailed grid located inside it were determined;

2) the found V_i elements were ordered in descending order of absolute permeability k_i;

3) in each V_i of the ordered set, the singular water saturation $s_i = 1$ was sequentially set before the condition $\sum_{i=1}^{N} m_i |V_i| \geq M|V|S$ was satisfied.

As an "accurate" solution for accuracy estimation, a solution based on a detailed grid in the entire deposit area was used, starting from the beginning of the reservoir development of 01.2000. The dimensions of the grid blocks were set in a manner similar to the detailed grid of the local refinement section – an average diameter of 15 m and with a breakdown of reservoir thickness for five final volumes according to the geometry of interlayers and bridges. The detailed grid of the entire deposit contained 92100 finite volumes. Simulation of the development of the entire deposit on a super element grid and on detailed grids was carried out until 01.2006.

The summary information on the used computational grids and computer time costs is given in Table 3. We note that in this paper we considered a model of a small oil deposit containing only 33 wells and a short period of its development (6 years). Therefore, the time expended on constructing a local refinement of the super element model in the section with 9 wells was only a few times shorter than the simulation time on the detailed grid of the deposit as a whole. Obviously, when considering large fields that number hundreds and thousands of wells and are being developed over several decades, the computational costs of building local and global models on detailed grids will differ by several orders of magnitude. For example, a grid with a lateral step of 15 m and a vertical step of 1 m covering an oil deposit of an average length of 5 km and an average thickness of 50 m will have a dimensionality of the order $5{\cdot}10^6$. In this case, the number of unknowns in solving the problem (1) on a small grid throughout the deposit area will increase by approximately 60 times. According to estimates of the dependence of the counting time on the dimension of the grid, we can conclude that the time required for constructing one option of the numerical solution in the entire area of such a deposit will increase approximately 100 times in comparison with the example considered, that is, about 15 days. But the dimension of the super element grid will increase only 5 times, and the duration of super element modeling will not exceed 2 minutes. The cost of the time for a performed only once upscaling will be about 10 hours. It should be noted that for large oil fields, the use of detailed computational grids may not be technically feasible at all or may require unacceptable costs of computational resources.

The time required for constructing a model on a super element grid is composed of three components – the performance of absolute permeability upscaling, relative phase permeability upscaling and modeling of water flooding directly (Table 3). The cost of modeling is less than 1%, since the upscaling procedures require solving a large number of auxiliary problems on detailed grids. On the other hand, upscaling is performed only once in the construction of the super element grid, so the use of super element model has significant advantages in carrying out multivariate design calculations in large oil fields. The greatest computational costs (more than 80%) fall on the procedure of relative phase permeability upscaling, which requires the solution of non-stationary two-phase flow problems for each super element. As shown in the work (Potashev, 2017), it is possible to accelerate the solution of this problem in principle by using an apparatus of artificial neural networks that use the statistical parameters of the local distribution of reservoir properties as input data.

3. Results of local refinement of the solution

Figure 5 shows the saturation distributions in the vertical section, plotted according to the detailed and super element model of the entire deposit at the time of the local refinement onset of super element model 07.2003. It can be seen that the super element grid allows us to describe the behavior of the average saturation, but does not give a detailed picture of the saturation distribution across the interlayers. The results of downscaling (transfer of the saturation field from super element to a detailed grid) are shown on the lower profile of Figure 5. It is possible to observe a completely satisfactory coincidence of the descaled field and the corresponding saturation distribution calculated on a detailed grid. The descaled saturation field was specified as the initial condition (2) for the local refinement task of the super element model.

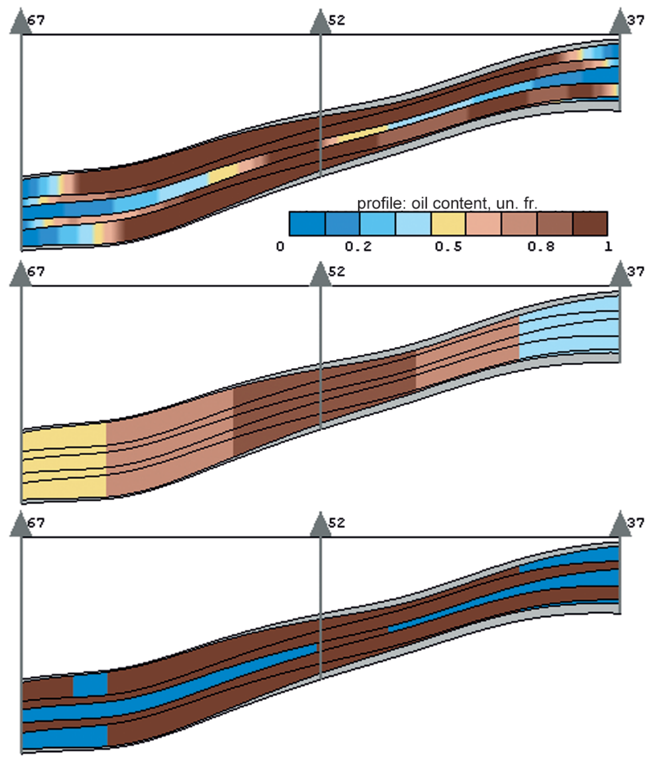

Figure 5. Distribution of saturation in the reservoir section as on 07.2003 for the detailed model of the whole deposit (above), for the super element model on a coarse grid (in the middle) and according to the descaling from the super element grid (bottom)

Computational grid	Lateral step, m	Number of layers vertically, un.	Dimensionality of grid, un.	Period of simulation, min	Computational time, min	
Super element on the entire deposit	225	1	90	01.00-01.06 гг.	upscaling (AP)	16
					upscaling (RPP)	87
					calculation	0.25
Detailed on the section	15	5	17 900	07.03-01.06 гг.	35	
Detailed on the entire deposit	15	5	92 100	01.00-01.06 гг.	216	

Table 3. Computational grid parameters and calculation duration

Calculations showed that over time, the agreement between the saturation fields calculated by the model of local refinement and the detailed model of the entire deposit is preserved to a certain extent (Figure 6).

Figure 7 shows the change in the pressure fields and the flow rate near the well 52, constructed on a detailed computational grid before and after the moment of filling the perforation interval.

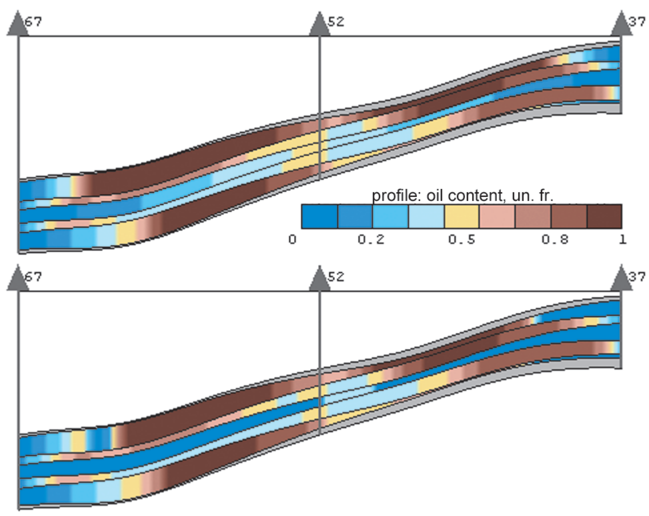

Figure 6. Distribution of saturation in the reservoir section as on 01.2006 for the detailed model of the whole deposit (above), for the super element model local refinement (bottom)

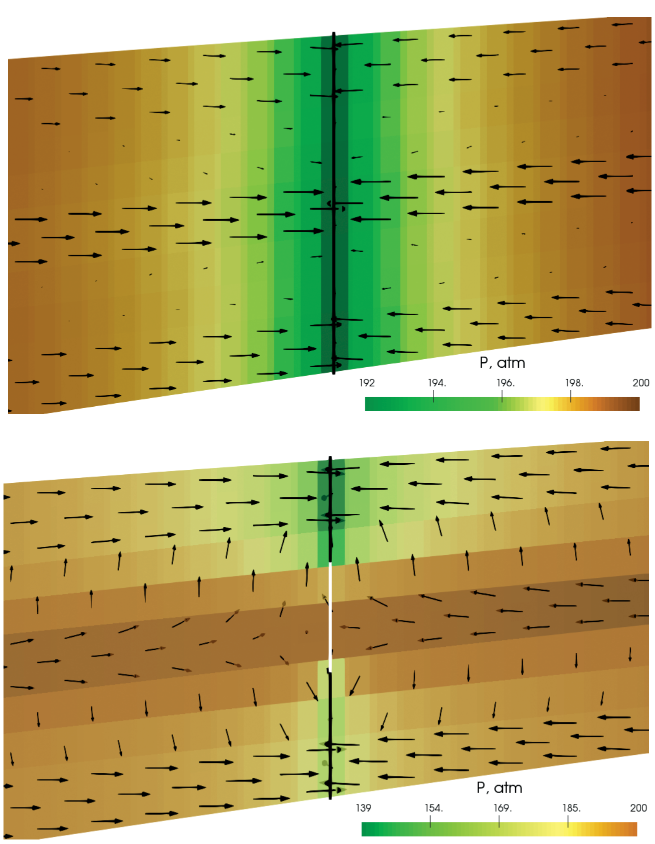

Figure 7. Distribution of reservoir pressure and filtrational rates in the vicinity of well 52: before (left) and after (right) isolating the middle section of the perforation interval

It is expected that the hydrodynamic consequences of the isolation of the highly permeable and the most watered interlayer will be as follows. Some time after pouring, the water cut in well 52 will be reduced due to the inflow of oil from the upper and lower less permeable layers to it. Then water will begin to flow into these layers from the isolated interlayer, which will lead to an increase in watering of the liquid taken out by the well. This behavior of oil selection was confirmed by calculations both from the detailed model of the entire reservoir and from the local refinement of the super element model (Figure 8). The curve obtained within the first stage of the super element model on a coarse grid describes only the average behavior of the true dynamics of the watering. But after a certain time of the effect of this geological and technical measure, the performance of the well, calculated on a coarse super element grid and on detailed grids, will converge.

As mentioned above, the local refinement of the super element grid also makes it possible to distribute the inflow to the well by individual intervals (interlayers). Figure 9 shows the flow rate of oil and water in well 52 for each of the three permeable

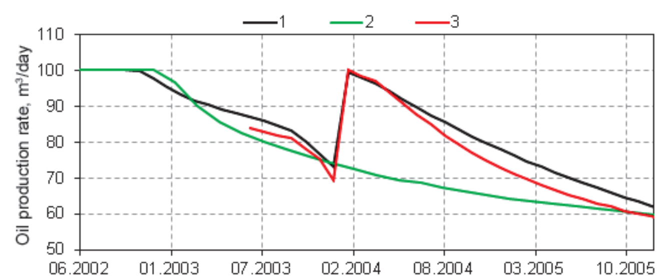

Figure 8. Estimated dynamics of oil production rate in well 52. 1 – by detailed model of the whole deposit, 2 – by super element model, 3 – by super element model with local refinement

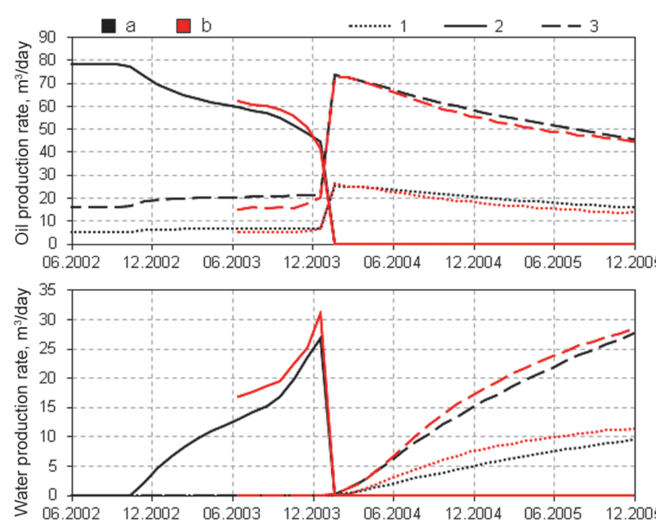

Figure 9. The calculated separation of the oil and water production rate from the first (1), the second (2) and the third (3) interlayers according to the detailed model of the whole reservoir (a) and the super element model with local refinement (b)

interlayers. Since the permeability of both bridges is lower by orders of magnitude, the fractions of the liquid inflow through them to the well are insignificant and not shown. It can be seen that the distribution of the inflow from the detailed model of the entire reservoir and the local refinement of the super element model are consistent with each other.

Conclusion

The presented method of two-stage super element modeling of an oil reservoir with local refinement of the solution is suitable for multivariate calculations in the design and evaluation of the efficiency of geological and technical measures in some parts of the oil field.

The advantage of this approach in comparison with the usual refinement of the computational grid in the entire flow area is clearly manifested when carrying out multivariate design calculations in large oil fields. Computational costs are determined by the preliminary stage of building a super element model and are weakly dependent on the number of sections selected to refine the solution and assess the consequences of geological and technical measures on the performance of wells. Moreover, the method of local refinement of the solution is applicable not only to modeling the redistribution of filtration flows by isolating individual intervals, but with minimal correction to other methods of oil recovery enhancement, for example, cyclic injection of the injected agent, acid treatment of the bottom-hole zone, sidetracking and other geological and technical measures on separate wells.

The accuracy of the results of the local refinement for the super element oil reservoir model is largely determined by the downscaling quality of the water saturation – the transfer of function S calculated on the coarse grid of super elements to the detailed grid constructed on the selected section of the reservoir. The simplest method of de-scaling S, adopted in the article, in determining the initial distribution of saturation s^0 needs to be improved. Considerable attention is expected to be given to this problem in the planning of prospective works.

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