ORIGINAL RESEARCH ARTICLE

DOI: https://doi.org/10.18599/grs.2018.2.108-114

Hydraulic fracturing efficiency evaluation in the vicinity of a single well for a reservoir with two fractures

Yu.I. Yakhina
Termosim LLC, Kazan, Russian Federation
E-mail: vib-210@mail.ru

Abstract. The solution of fluid flow problem in an unbounded homogeneous reservoir penetrated by a single well with two intersecting fractures with taking into account their hydrodynamic resistance is constructed and investigated. A general integral representation of the perturbed pressure field is obtained using the instantaneous point-source. As a particular case, the quasi-stationary operating mode of the well is considered. The accuracy and limitations of the obtained asymptotic solutions are estimated. A comparative analysis obtained results is done. Well productivity and the pseudoskin factor for the fractures system are determined, explicit analytical expressions for these characteristics are constructed. In the course of computational experiments, the interaction of intersecting fractures at different opening angles are investigated. An estimate of the efficiency of repeated hydraulic fracturing of the productive formation is obtained. It is shown that the maximum flow is achieved for a perpendicular arrangement of the fractures, and the distribution of the outflow (inflow) along the flat vertical fracture essentially depends on its relative filtration resistance.

Keywords: hydraulic fracturing, crack azimuth of repeated fracturing, efficiency of repeated fracturing, quasi-stationary operation of well, pseudoskin factor of repeated fracturing, filter resistance in cracks, instantaneous point-source, fracture hydrodynamics

Recommended citation: Yakhina Yu.I. (2018). Hydraulic fracturing efficiency evaluation in the vicinity of a single well for a reservoir with two fractures. *Georesursy = Georesources,* 20(2), pp. 108-114. DOI: https://doi.org/10.18599/grs.2018.2.108-114

Introduction

In order to involve undrained and weakly drained mineral resources in development, hydraulic fracturing is used. This technology contributes to the creation of a high-conductive fracture, which allows increasing the productivity of producing wells or injectivity of injection wells. According to expert assessments of hydrodynamic studies, the hydraulic fracturing of the reservoir is the most effective geological and technical measure that guarantees an increase in the efficiency of poorly permeable reservoirs development. One of the first studies of stationary single-phase inflow to a well with a single fracture of hydraulic fracturing is presented in the paper (Prats, 1961). Theoretical calculations show that the fracturing allows several times to increase the well production by activating the weakly draining zones, the volume of extraction or injection depending on the conductivity and fracture length (Cinco-Ley et al., 1978; Meehan et al., 1989).

A review of a number of other publications and an analysis of the efficiency of fracturing in a well with a single vertical fracture in a circular feed loop was carried out (Morozov, 2016) on the basis of various estimates of productivity and the generalized pseudoskin factor concept (Economides et al., 2002).

With a significant reduction in the production rates with respect to the initial regimes, a second fracturing is performed. At the same time, to ensure full coverage of the reservoir and introduction of new reserves, a method of reorientation of the azimuth is used, which makes it possible to realize the development of the fracture in a direction different from the first hydraulic fracturing.

Repeated hydraulic fracturing is one of the most common methods of intensifying the extraction of mineral reserves and increasing reservoir productivity, which makes the task of analyzing and predicting its effectiveness topical. In a particular case, the problem of a stationary inflow to a well with several vertical fractures with uniformly distributed inflow along the fractures in the region bounded by the circular feed circuit was considered in the work (Raghavan, Joshi, 1993). A similar problem of a steady uniformly distributed inflow to two fractures with different geometric characteristics at a given intersection angle was solved numerically in (Lihtarev, Pestrikov, 2010).

In the framework of this study, theoretical issues related to the analysis of the unsteady hydrodynamic regime of one and two intersecting fractures of different azimuth are considered taking into account their hydraulic resistance in an unbounded homogeneous reservoir. The efficiency of re-fracturing at different opening angles was estimated. Explicit analytical expressions are obtained for the productivity of the well and the pseudoskin factor.

General integral representation of the disturbance in the reservoir pressure field

During the operation of the well, fluid movement in the near-wellbore zone occurs due to the potential energy of the reservoir elastic state, as a result of lowering pressure at the bottom of the well. At the same time, the volume of liquid increases, and the porosity, and possibly the permeability, decreases due to the expansion of the formation skeleton. In the case of fluid injection in the formation, the flow of injected fluid is maintained by increased pressure at the bottom, and the filtration process develops in the opposite direction.

The main characteristic of filtration processes is the field of reservoir pressure, which, in the case of unsteady elastic filtration in a porous medium with distributed sources, is described by the piezoconductivity equation (Charnyy, 1963):

$$\frac{\partial P}{\partial t} = \varkappa \Delta P + \frac{q^*}{m_0 \beta}; \quad |x|, |y|, |z| < \infty, t > 0,$$

where $\varkappa = k/(\mu\beta m_0)$ is the conductivity coefficient, which determines the propagation velocity of pressure perturbations $P(x,y,z,t)$ in a porous medium with permeability k, porosity m_0, elastic capacity coefficient β, liquid viscosity μ; $q^*(x,y,z,t)$ is the density of distributed sources, which is the amount of fluid entering the volume unit of the medium per unit time t; Δ is the differential Laplace operator in a system of Cartesian coordinates (x,y,z).

Let there be no perturbation of pressure in the reservoir at the initial moment of time, then the solution of the piezoconductivity equation for zero initial conditions can be written in a general integral form using the method of instant point sources (Carslaw, Jaeger, 1964; Tikhonov, Samarskii, 1999):

$$P(x,y,z,t) = \int_0^t \iiint_{-\infty}^{+\infty} \frac{q^*(\xi,\eta,\zeta,\tau)}{m_0\beta} G^*(\xi,\eta,\zeta,x,y,z,t-\tau)\, d\xi\, d\eta\, d\zeta\, d\tau, \quad (1)$$

where $G^*(\xi,\eta,\zeta,x,y,z,t-\tau)$ is the influence function of the instantaneous point source, which is the pressure perturbation at the point (x,y,z) at time t, caused by a single instantaneous point source at the point (ξ,η,ζ) at time $t = \tau$:

$$G^*(\xi,\eta,\zeta,x,y,z,t-\tau) = \left(\frac{1}{2\sqrt{\pi\varkappa(t-\tau)}}\right)^3 e^{-\frac{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}{4\varkappa(t-\tau)}}.$$

The total inflow or outflow of fluid Q in a well with fractures per unit time is, by definition, equal to:

$$Q(t) = \iiint_{-\infty}^{\infty} q^*(\xi,\eta,\zeta,t)\, d\xi\, d\eta\, d\zeta.$$

We use the obtained representation (1) to study the filtration process when liquid is injected in a homogeneous isotropic reservoir with a thickness h in the vicinity of an injection well with two flat vertical fractures created as a result of hydraulic fracturing and intersecting at an angle α (Fig. 1). In order to simplify further reasoning, we assume that fractures of length $2x_f$ and width δ_f are symmetrically disposed about the axis of the well, have the same geometry and identical hydrodynamic properties. The flow rate $Q(t)$ pumped into the well is distributed evenly along the thickness of the reservoir (fracture height) with the surface density $q(x,t)$. Here, q is the amount of liquid injected per unit of formation thickness from a unit of fracture length per unit time. The density of the distributed source along the fracture, taking into account the smallness of the opening of the discontinuity, δ_f, is determined by the relation: $q^*(x,y,z,t) \approx q(x,t)/\delta_f$ (at $q^* = 0$ outside the fracture $|y| > \delta_f/2$).

In view of the linearity of the problem under consideration and the identity of the fractures , the perturbation of the pressure field created by them can be regarded as the sum of the perturbations caused by single fractures , one of which is located along the x axis as shown in Fig. 1, and the other, a repeated fracture , is directed along the axis with coordinates:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Assuming homogeneity of the reservoir and uniformity of vertical injection, the general integral representation of the pressure field (1) after integration over η is transformed to the form:

$$P(x,y,t) = \frac{\mu}{4\pi k} \int_0^t \int_{-x_f}^{x_f} \frac{q(\xi,\tau)}{(t-\tau)} \left[e^{-\frac{(x-\xi)^2+y^2}{4\varkappa(t-\tau)}} + e^{-\frac{(x'-\xi)^2+y'^2}{4\varkappa(t-\tau)}} \right] d\xi\, d\tau. \quad (2)$$

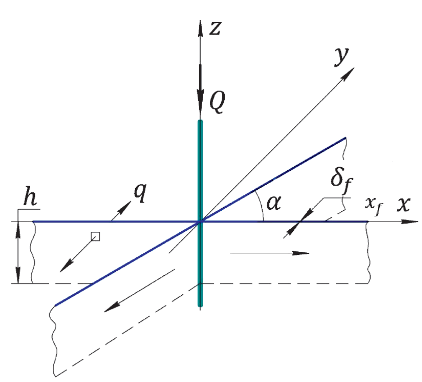


Fig. 1. Injection well with suppressing flat fractures

The total flow rate Q takes the form:

$$Q(t) = 2h \int_{-x_f}^{x_f} q(x,t)\,dx.$$

Single fracture hydrodynamics

Hydraulic fracturing of the productive formation allows creating a fracture with a high permeability $k_f >> k$, which leads to a decrease in the hydraulic resistance of the bottomhole zone and, as a consequence, to an increase in the filtration surface. For the same reason, the transient processes of pressure redistribution in the Pf fracture occur very rapidly, and the fluid flow at each instant of time can be described by a stationary filtration equation (Meehan et al., 1989):

$$\frac{k_f \delta_f}{\mu} \frac{\partial^2 P_f}{\partial x^2} = q;\; 0 < x < x_f, t > 0. \quad (3)$$

By assumption, the homogeneous fracture consists of two differently directed «wings» symmetrical with respect to the well, which allows us to consider the liquid filtration region in only one direction of the fracturing (for example, in the interval $0 < x < x_f$), as shown in Fig. 2.

At the bottom of the well, there is a perturbation of the fluid pressure P_w, relative to the reservoir, and the inflow from the end is considered negligible. Thus, equation (3) is supplemented by boundary conditions:

$$P_f(x,t)|_{x=0} = P_w,\; \left.\frac{\partial P_f}{\partial x}\right|_{x=x_f} = 0.$$

The solution of the problem formulated above, which describes the process of filtration in a fracture, can be represented in the form:

$$P_f(x,t) = P_w - \frac{\mu}{k_f \delta_f}\left[\int_0^x \xi q(\xi,t)d\xi + x\int_x^{x_f} q(\xi,t)d\xi\right],$$

$0 < x < x_f$. (4)

Relation (4) relates the pressure distribution along the fracture to the liquid flow density $q(x,t)$, and should be considered together with equation (2) at $y = 0$:

$$P_f(x,t) = \frac{\mu}{4\pi k}\int_0^t \int_{-x_f}^{x_f} \frac{q(\xi,\tau)}{(t-\tau)}\left[e^{-\frac{(x-\xi)^2}{4\varkappa(t-\tau)}} + e^{-\frac{(x'-\xi)^2+y'^2}{4\varkappa(t-\tau)}}\right]d\xi\,d\tau, \quad (5)$$

where $0 < x < x_f$, $t > 0$, $x' = x\cos\alpha$ and $y' = x\sin\alpha$.

To determine the distributed inflow along the fracture, combining equations (4) and (5), we obtain the integral equation:

$$P_w = \frac{\mu}{4\pi k}\int_0^t \int_{-x_f}^{x_f} \frac{q(\xi,\tau)}{(t-\tau)}\left[e^{-\frac{(x-\xi)^2}{4\varkappa(t-\tau)}} + e^{-\frac{(x'-\xi)^2+y'^2}{4\varkappa(t-\tau)}}\right]d\xi\,d\tau +$$
$$+ \frac{\mu}{k_f \delta_f}\left[\int_0^x \xi q(\xi,t)d\xi + x\int_x^{x_f} q(\xi,t)d\xi\right], \quad (6)$$

where $0 < x < x_f$ and $t > 0$.

Quasistationary well regime with intersecting fractures

As preliminary estimates show, under unchanged operating conditions of the well (constant bottomhole pressure P_w or constant flow rate Q), the quasistationary filtration regime in the reservoir, characterized by the time-invariant profile of the relative distribution of the inflow along the fracture

$$\overline{q}(x) = q(x,t)/< q >,$$

where $< q >= Q(t)/(4hx_f)$ is set at $\varkappa t/x_f^2 \geq 100$.

In this case, the integral equation (6) takes the form:

$$P_w = -\frac{\mu Q(t)}{4hx_f}\left\{\frac{1}{4\pi k}\int_0^{x_f} \overline{q}(\xi)\left[Ei\left(-\frac{(x+\xi)^2}{4\varkappa t}\right) + \right.\right.$$
$$+ Ei\left(-\frac{(x'+\xi)^2+y'^2}{4\varkappa t}\right) + Ei\left(-\frac{(x-\xi)^2}{4\varkappa t}\right) +$$
$$\left. + Ei\left(-\frac{(x'-\xi)^2+y'^2}{4\varkappa t}\right)\right]d\xi +$$
$$\left. + \frac{1}{k_f\delta_f}\left[\int_0^x \xi\,\overline{q}(\xi)\,d\xi + x\int_x^{x_f}\overline{q}(\xi)\,d\xi\right]\right\}. \quad (7)$$

Here $Ei(-x) = -\int_x^\infty \frac{e^{-u}}{u}du$ – is integral exponential function, for small values of the argument (Abramovitz, Stegun, 1979):

$$Ei(-x) \approx \ln(1.781x).$$

Accordingly, in the quasi-stationary approximation, instead of (7) for large values of time t, we have:

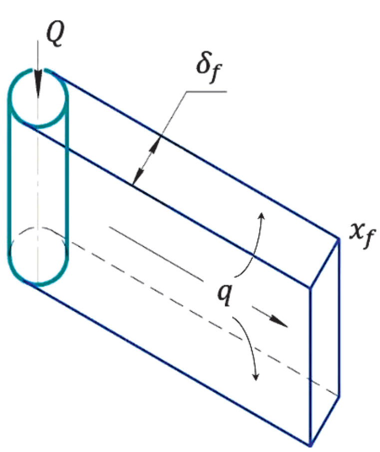


Fig. 2. Geometrical model of a wing of a fracture

$$P_w = -\frac{\mu Q(t)}{4hx_f}\left\{\frac{1}{4\pi k}\int_0^{x_f}\overline{q}(\xi)\left[ln\left(\frac{(x+\xi)^2}{2.2458\,\varkappa t}\right)+\right.\right.$$

$$+\,ln\left(\frac{(x-\xi)^2}{2.2458\,\varkappa t}\right)+ln\left(\frac{(x'+\xi)^2+y'^2}{2.2458\,\varkappa t}\right)+$$

$$\left.+\,ln\left(\frac{(x'+\xi)^2+y'^2}{2.2458\,\varkappa t}\right)+ln\left(\frac{(x'-\xi)^2+y'^2}{2.2458\,\varkappa t}\right)\right]d\xi+$$

$$\left.+\,\frac{1}{k_f\delta_f}\left[\int_0^{x}\xi\,\overline{q}(\xi)\,d\xi + x\int_x^{x_f}\overline{q}(\xi)\,d\xi\right]\right\}, \qquad (8)$$

where $x' = x\ cos\alpha$ и $y' = x\ sin\alpha$.

The numerical solution of the distributed flow rate of liquid along a fracture in the presence of a filtration resistance

Here and below, the following typical values for the injection conditions of the liquid (water) are accepted in the calculations: the pressure at the bottom is $P_w = 4.053\cdot10^6$ Pa (40 atm), the fracture half-length is $x_f = 100$ m, the fracture width is $\delta_f = 0.2$ m, the piezoconductivity coefficient is $\varkappa = 2.78$ m^2/s (10^4 m^2/hour), the permeability of the reservoir is $k = 10^{-13}$ m^2 (100 mD), the viscosity of the fluid is $\mu = 0.001$ Pa·s (1 cps), the operating time of the well is t = 360000 sec (100 hours).

The results of the computational experiments presented in Fig. 3a have shown that the distribution of the inflow along the fracture essentially depends on its relative filtration resistance, $kx_f/(k_f\delta_f)$. At the end of the fracture , a sharp increase in the outflow is observed, which is associated with a change in the character of the liquid flowfrom the linear to the quasiradial one.

In turn, for a known injection density $q(x,t)$, which takes into account the filtration resistance in the fractures, the quasi-stationary pressure distribution in the reservoir can be calculated on the basis of the asymptotic representation of the general equation (2):

$$P(x,y,t) = -\int_0^{x_f}\frac{\mu q(\xi,t)}{4\pi k}\left[Ei\left(-\frac{(x+\xi)^2+y^2}{4\varkappa t}\right)+\right.$$

$$+\,Ei\left(-\frac{(x-\xi)^2+y^2}{4\varkappa t}\right)+Ei\left(-\frac{(x'+\xi)^2+y'^2}{4\varkappa t}\right)+$$

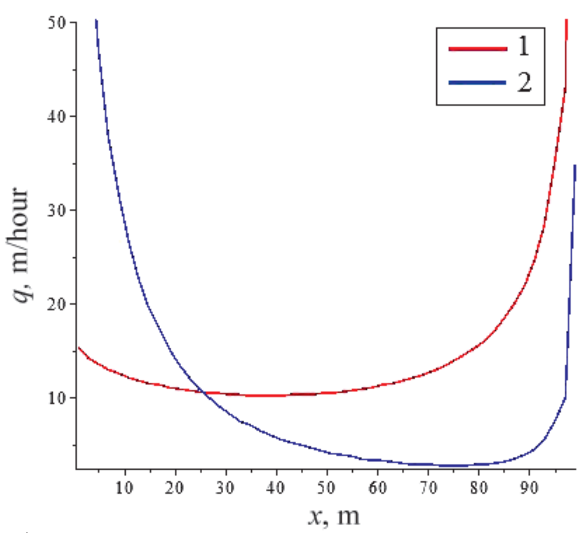
$$\left.+\,Ei\left(-\frac{(x'-\xi)^2+y'^2}{4\varkappa t}\right)\right]d\xi,$$

where $x' = x\ cos\alpha - y\ sin\alpha$ and $y' = x\ sin\alpha + y\ cos\alpha$.

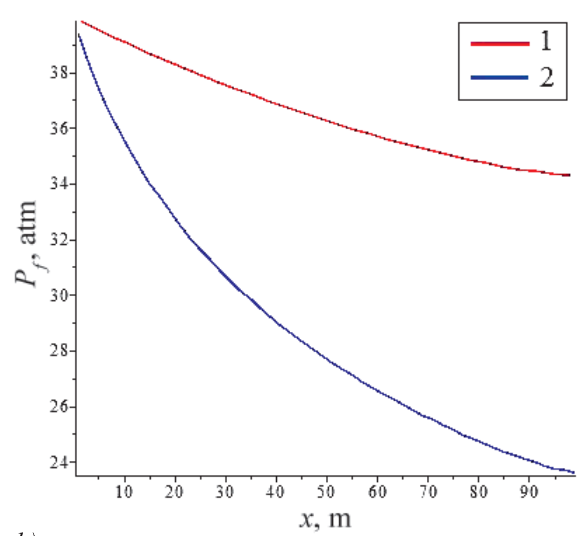
A comparative analysis (Fig. 4) shows that the pressure drop along the fractures increases substantially with increasing filtration resistance.

Let us consider the distribution of the inflow into the reservoir with a uniform pressure distribution along the fractures , $P_f = P_w$, in the absence of filtration resistance ($k_f \rightarrow \infty$). An analysis of the solution of the inverse problem of the outflow distribution at a given pressure shows that when the opening angle between fractures increases, the pressure gradients in the near wellbore zone of the reservoir decrease and the local outflow of liquid decreases, as shown in Fig. 5. And at the remote end part of the fracture , the reverse effect is observed – the flow density increases due to the decrease in interference between the fractures.

When the opening angle tends to 0°, the pressure in the well near the wellbore is equalized and the injection density is correspondingly reduced – two fractures begin to work as one and the outflow from the central part increases.



a)



b)

Fig. 3. The distribution of flows to the fracture (a) and the pressure along the fracture (b) in the vicinity of the well at a mutually perpendicular arrangement of fractures and different permeabilities: 1 – $k_f = 5\cdot10^4$ mD and 2 – $k_f = 10^4$ mD.

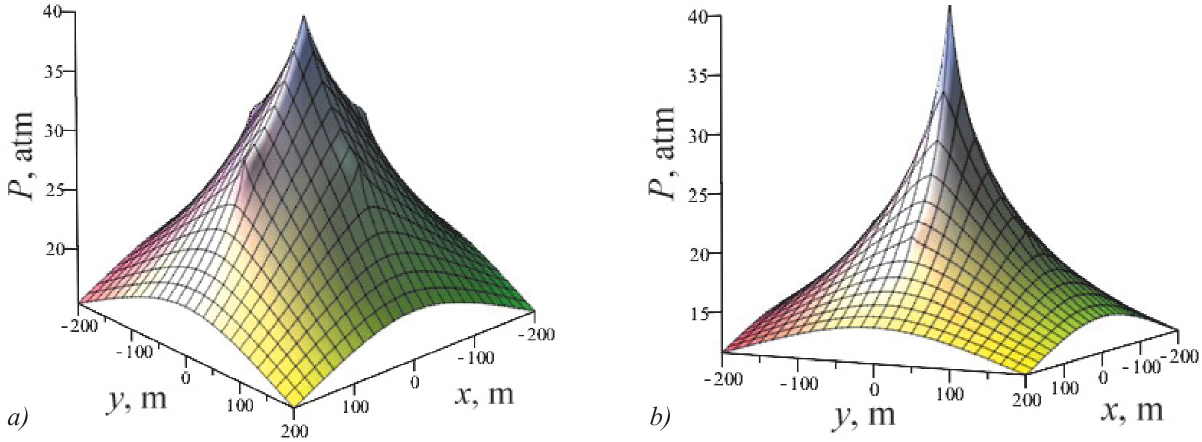


Fig. 4. Perturbation of reservoir pressure in the vicinity of the well with mutually perpendicular arrangement of fractures with permeability: (a) – $k_f = 5{\cdot}10^4$ mD ($kx_f/(k_f\delta_f) \sim 1$) and (b) – $k_f = 10^4$ mD ($kx_f/(k_f\delta_f) \sim 5$)

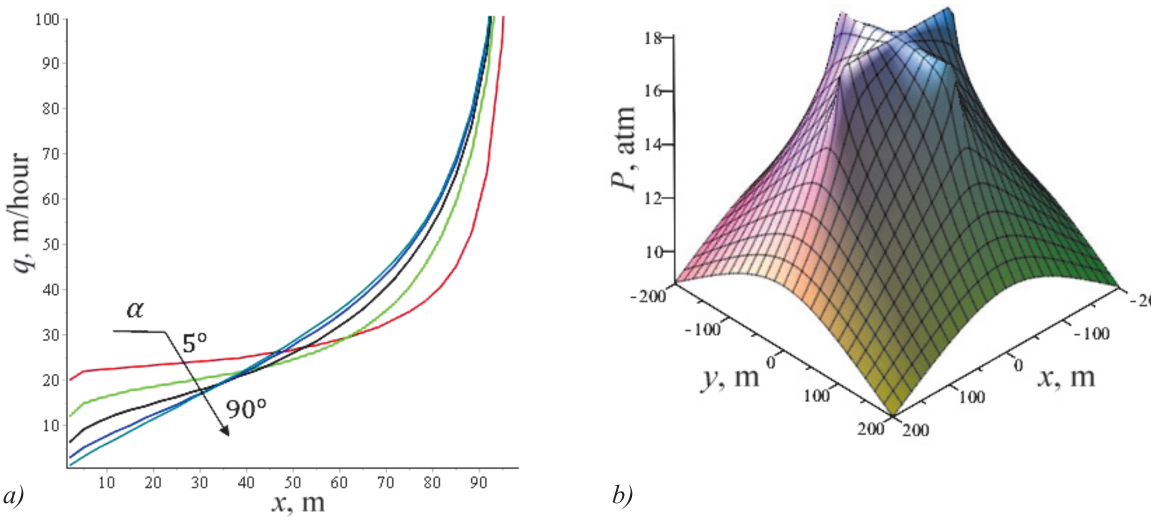


Fig. 5. Distribution of the inflow along the primary fracture (a) at a constant depression of 40 atm at different opening angles and perturbation of the reservoir pressure (b) in the vicinity of the well with mutually perpendicular arrangement of the fractures

Well productivity and pseudoskin factor

Summarizing the traditional concept of the skin factor (Economides et al., 2002; Morozov, 2016), we define the pseudoskin factor of the well with intersecting fractures of the finite permeability for a given depression. From the relation (8) for $x = 0$ it follows immediately:

$$P_w = -\frac{\mu Q(t)}{4hx_f}\left[\int_0^{x_f}\frac{2\bar{q}(\xi)}{\pi k}\,ln\left(\frac{\xi}{\sqrt{2.2458\varkappa t}}\right)d\xi\right] =$$

$$= -\frac{\mu Q(t)}{2\pi k h x_f}\left\{\int_0^{x_f}\bar{q}(\xi)\left[ln\left(\frac{\xi}{r_w}\right) - ln\left(\sqrt{\frac{2.2458\varkappa t}{r_w^2}}\right)\right]d\xi\right\},$$

where r_w is the radius of the well. As a result, the productivity of the well will be given by the formula:

$$\frac{Q}{P_w} = \frac{2\pi hk}{\mu}\frac{1}{S + ln\left(\sqrt{\frac{2.2458\varkappa t}{r_w^2}}\right)}, \qquad (9)$$

where pseudoskin factor is:

$$S = -\frac{1}{x_f}\int_0^{x_f}\bar{q}(\xi)\,ln\left(\frac{\xi}{r_w}\right)d\xi.$$

In the particular case of a single fracture of infinite conductivity ($\bar{q} = 1$) the pseudoskin factor in (9) is equal to $S = -ln(x_f/e)$. In addition, according to the estimates obtained in (Charnyy, 1963), on the basis of the method of successive change of stationary states, the expression $\sqrt{2.2458\varkappa t}$ can be considered as the radius of the mobile current circular feed loop r_e. As a result, the above relation (9) can be rewritten in the equivalent form:

$$P_w = \frac{\mu Q}{2\pi kh}\,ln\left(\frac{r_e}{x_f/e}\right),$$

which coincides with the solution (Raghavan, Joshi, 1993).

Calculation results of pseudoskin factor (9), shown in Fig. 6, show that the minimum pseudoskin factor and the maximum injection (inflow) volume increase is achieved with the perpendicular arrangement of fractures and essentially depends on their resistance to filtration.

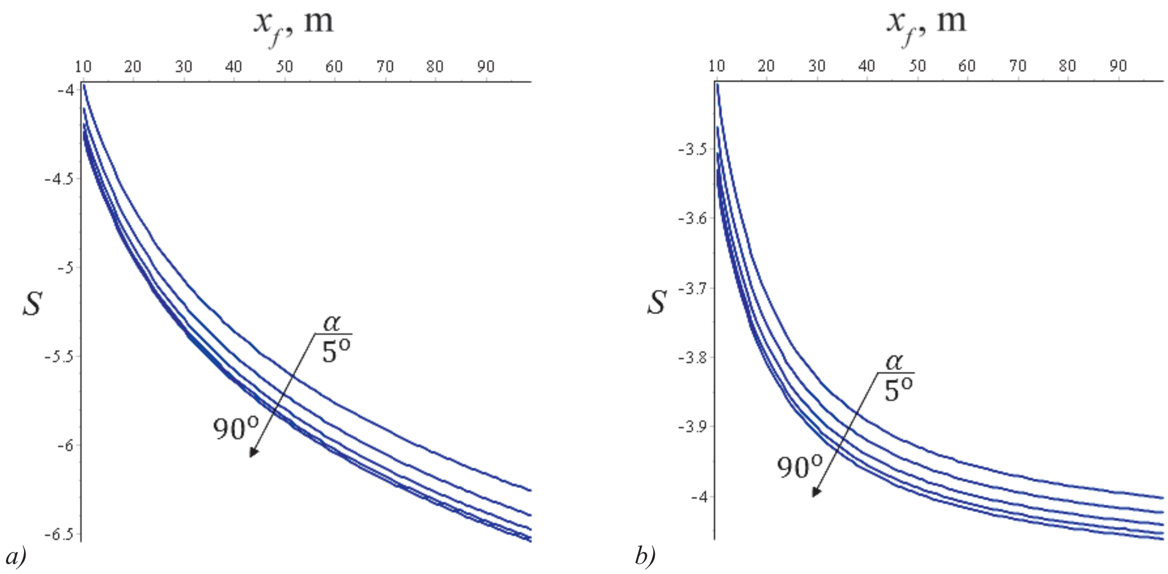


Fig. 6. Pseudoskin factor at a given bottomhole pressure P_w with permeability (a) $k_f = 10^6$ mD ($kx_f / (k_f\delta_f) \sim 0{,}05$) and (b) $k_f = 10^4$ mD ($kx_f / (k_f\delta_f) \sim 5$)

Repeated hydraulic fracturing allows to increase the pseudoskin factor at high fracture permeability up to 5-10%, as shown in Fig. 6a. Efficiency of re-fracturing according to Fig. 6b increases to 10-20% in the case of small fractures permeability, which is in agreement with the numerical solutions obtained in (Lihtarev, Pestrikov, 2010).

Conclusion

As is known, hydraulic fracturing of the reservoir is a complex, energy-intensive and expensive technological process for intensifying field development. Every year, this technology is increasingly used in the operation of both producing and injection wells. However, the consequences of applying repeated fracturing are not always positive, which makes the task of assessing its effectiveness urgent.

In the study of the hydrodynamic regime of a well with intersecting fractures penetrating a homogeneous reservoir, it is shown that under unchanged conditions of well operation (steady pressure at the bottom P_w or constant flow rate Q), a quasi-stationary operation mode of the well with a constant injection profile (inflow) in fractures is established. The distribution of the inflow along the fracture essentially depends on its relative filtration resistance, $kx_f/(k_f\delta_f)$. At the same time, as the opening angle is increased, the pressure gradients in the near wellbore zone of the reservoir decrease and the local outflow (inflow) of the liquid decreases. At the remote end part of the fractures , the reverse effect is observed – the flux density increases due to the decrease in interference between the fractures. The maximum total increase in inflow into the reservoir is achieved with a perpendicular arrangement of the fractures .

Repeated hydraulic fracturing of the productive formation allows to increase the pseudoskine factor at a high permeability ($k_f \sim 10^6$ mD, $kx_f/(k_f\delta_f) \sim 0{,}05$) fractures up to 5-10%. The effectiveness of re-fracturing increases to 10-20% in the case of small permeability ($k_f \sim 10^4$ mD, $kx_f/(k_f\delta_f) \sim 5$).

Further research continuation can be aimed at solving both more general problems of unsteady filtration and inverse problems: justifying the methods of hydrodynamic exploration of wells in order to determine the opening angle between two hydraulic fracturing fractures and other hydrodynamic characteristics of the fracture system.

Acknowledgements

The author expresses gratitude to the scientific adviser, Professor A.N. Salamatin for valuable advice in the planning and implementation of research and recommendations on the formatting of work. The author thanks Professor G.T. Bulgakova for choosing the topic and area of research and expresses gratitude to the reviewer for valuable comments and suggestions that contributed to the improvement of this article.

References

Abramowitz M., Stegun A. (1979). Handbook of Mathematical Functions. Moscow: Nauka, pp. 58-59. (In Russ.)

Carslaw H.S, Jaeger J.C. (1964). Conduction of Heat in Solids. Moscow: Nauka, pp. 251-259. (In Russ.)

Charnyy I. A. (1963). Underground hydro gas dynamics. Moscow: Gostoptekhizdat, pp. 250-313. (In Russ.)

Cinco-Ley H., Samaniego V.F., Domínguez A.N. (1978). Transient pressure behavior for a well with a finite-conductivity vertical fracture. *SPE Journal,* 18(4)*,* pp. 253-264.

Economides M., Oligney R., Valkó P. (2002). Unified fracture design: bridging the gap between theory and practice. Alvin, Texas: Orsa Press, 25 p.

Lihtarev A. V., Pestrikov A. V. (2010). Opredelenie kojefficienta produktivnosti skvazhiny, peresechennou dvumua treshhinami gidrorazryva plasta raznogo azimuta, na osnove matematicheskogo modelirovaniua [Determination of the production efficiency of a well crossed by two induced hydraulic fractures of a different azimuth, based on mathematical modeling]. *Nauchno-tehnicheskiy vestnik OAO " NK" Rosneft'"*, pp. 12-14. (In Russ.)

Meehan D. N., Horne R.N., Ramey H.J. (1989). Interference testing of finite conductivity hydraulically fractured wells. *SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers,* pp. 137-152.

Morozov P.E. (2016). Psevdoskin-faktor i optimal'naya provodimost' vertikal'noy treshhiny gidavlicheskogo razryva plasta [Pseudoskine factor and optimal conductivity of vertical induced hydraulic fracture]. *Mezhd. nauchno-prakt. konf.: Innovacii v razvedke i razrabotke neftyanyh i gazovyh mestorozhdeniy* [Proc. Sci. and Pract. Conf.: Innovations in explorationa and development of oil and gas fields], Kazan, pp. 53-56. (In Russ.)

Prats M. (1961). Effect of vertical fractures on reservoir behavior-incompressible fluid case. *SPE Journal*, 1(02), pp. 105-118.

Raghavan R., Joshi S.D. (1993). Productivity of multiple drainholes or fractured horizontal wells. *SPE Form. Eval.,* 8(01), pp. 11-16.

Tikhonov A. N., Samarskii A. A. (1999). Equations of Mathematical Physics. Moscow: MSU, pp. 477-485. (In Russ.)

About the Author

Yulia I. Yakhina – Junior Software Engineer, Engineering-Analytical Group, Department of Scientific Research, Termosim LLC

59/1 Magistralnaya str., Kazan, 420108, Russian Federation

E-mail: vib-210@mail.ru

Manuscript received 29 January 2018;
Accepted 28 April 2018;
Published 30 June 2018